

## THE USE OF REALISTIC MATHEMATICS EDUCATION IN TEACHING THE CONCEPT OF EQUALITY

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### ABSTRACT

Algebra is one of the most fundamental concepts in learning mathematics and the concept of equality is an essential part of learning algebraic topics. However, many students find this concept difficult to understand. The problem is the difficulty of the content and how to teach it. In order to solve the problem, this paper reviews the use of the theory of Realistic Mathematics Education (RME) in teaching the concept of equality. RME is a process of teaching and learning mathematics in a contextual way as mathematics is considered as a human activity. This paper aims to review the use of the RME theory in teaching the concept of equality within the Biblical perspective as the parameter of the implementation for mathematics as one of God's tools to reveal His glory and wisdom. The method used in this paper was literature review which used literature findings as the resources. According to the literature research, it was found that the main problem in teaching and learning the concept of equality was students' misconception of equality. The use of the RME theory can help teachers to teach the "equality" concept to help student learn contextually.

**Keywords:** The concept of equality, algebra, realistic mathematics education, RME

### ABSTRAK

Aljabar merupakan salah satu konsep paling dasar dalam pembelajaran matematika mendalam dan konsep persamaan merupakan bagian yang penting dalam mempelajari topik-topik aljabar. Namun ada murid-murid yang mendapati bahwa konsep persamaan sulit dimengerti. Permasalahannya adalah seputar sulitnya materi yang dipelajari dan cara pengajarannya. Untuk mencari solusi dari permasalahan tersebut, makalah ini akan mengkaji penggunaan teori Matematika Realistis dalam pengajaran konsep persamaan. Matematika Realistis adalah proses belajar mengajar matematika kontekstual yang memandang matematika sebagai aktifitas manusia. Makalah ini bertujuan untuk mengkaji penggunaan teori Matematika Realistis dalam pengajaran konsep persamaan di dalam perspektif Alkitabiah sebagaimana matematika adalah alat Allah untuk menyingkapkan kemuliaan dan hikmat-Nya. Metode yang digunakan dalam makalah ini adalah studi literatur yang menggunakan temuan-temuan literatur sebagai sumber. Berdasarkan studi literatur, masalah utama dalam pengajaran konsep persamaan adalah adanya miskonsepsi pada pemahaman murid dan menggunakan teori Matematika Realistis, pengajar dapat mengajar konsep "keseimbangan" untuk membantu murid belajar secara kontekstual.

**Kata kunci:** Konsep persamaan, aljabar, Pendidikan matematika realistik, RME

### INTRODUCTION

The concept of equality is an essential part of learning Algebraic topics and Algebra is one of the most fundamental concepts in learning mathematics for further mathematics lesson. However, there are many students who find it difficult to understand to concept of equality. One of the cause is that students have their own obstacles in learning mathematics such as incomplete or insufficient prior knowledge, lack of interest, pessimism and many

other problems difficult. At the end, students understand nothing other than the surface of the topic they are learning. Another factor is that in this era, many teachers are still using the traditional approach in teaching mathematics. Rossnan (2006) believes that the teaching of mathematics is generally done in the traditional method which revolves around the process of “memorizing” the mathematics concept.

There have been many changes as a result of many mathematics-related research in viewing mathematics as part of human living. One of the most popular findings is the method to treat mathematics from realistic interactions, realistic inventions, and realistic situations. This idea is summarized in the Theory of Realistic Mathematics Education (RME). Gravemeijer & Doorman (1999, pp. 116-117) stated that in RME, context problems are the basis for progressive mathematization, and through mathematizing, the students develop informal context-specific solution strategies from realistic situations.

God has called some of His people to be Mathematics teachers to help the younger generation get to see His glory in Mathematics. Teaching Mathematics from a Christian perspective is not just attaching some Bible verses in a lesson. Teaching mathematics in Christian perspective is delivering every single topic by letting the students see God’s providence in His creation, anywhere, anytime through learning from reality. From learning through real situations in their environments, students will see that mathematics is everywhere and learning it is important.

This paper will provide a literature review involving 5 main focus; Secondary Mathematics Topic: The Concept of Equality, The Obstacles in Teaching The Concept of Equality, The Theory of Realistic Mathematics Education (RME), The Principles of the Theory of RME, and Christian Perspective Concerning the Theory of RME. The purpose of this paper is to examine the use of the Theory of RME in teaching the concept of equality.

## **THE CONCEPT OF EQUALITY AND REALISTIC MATHEMATICS EDUCATION**

### **The Concept of Equality**

Cai, et al., stated that algebra has been characterized as an important ‘gatekeeper’ in mathematics (2005). Algebra controls the access to mathematics and that explains how important it is for teachers to deeply teach the essential concept of it to the students. Since 1557, the term “equal” and its symbol “=” have been present in several usage including mathematics. It was firstly used in by Robert Recorde in his “The Whetstone of Witte” printed books in 1557 (Cajori, 1930). Looking at today the equal sign is present in all stages of mathematics. The symbol “=” is used in mathematics as a very essential symbol for mathematical operations, especially in arithmetic operations towards algebra operations. In algebra, it is presented with the existence of variables in order to show certain value in a mathematical expression. For example, For example, “ $2x + 2 = 6$ ” is a statement about  $x$ , which can be translated into the statement “ $x = 2$ ”. Without a deep understanding of the concept of equality and equal sign, it might be impossible for students to understand how the mathematics symbol works and what it expresses. Knowing that the concept of equality is very important in algebraic topics then it has to be deeply understood by the students in the teaching of pre-algebra or the introduction of Algebra.

### **The Obstacles in Teaching The Concept of Equality**

There are basically two synthesized obstacles faced by both teachers and students when it comes to the teaching and learning the concept of equality.

### 1) Misconception

There have been many studies aiming to know the reason behind the difficulties faced by the students in the concept of equality. According to (McNeil et al., 2006), numerous research studies support the idea that students' required understanding of the equal sign can be placed into two basic categories:

- relational, the notion that the equal sign means that the two expressions on either side refer to the same quantity; or
- operational, the notion that the equal sign means "find the total" or the "and the answer is..."

However students often understand the equal sign merely as a place to put the final answer next to. Many students recognize the function of equal sign is only to represent a tool believing that it indicates where to write an answer (Knuth et al. 2008). Students often miss the important role of equal sign to express that both sides have the same value. This misconception of equal sign leads them to misconception in understanding the concept of equation in Algebra. Students often miss the relational understanding of equality. Students have to understand that the concept of equality is not just about the final answer and result of the operations but it is a relationship that two mathematical expressions stand for the same value. This misconception can cause further lack of understanding in future lessons. Limited conception of what the equal sign means is one of the major resistance in learning algebra for all manipulations on equations require understanding the equal sign as a relation (Carpenter, Franke, & Levi, 2003). Students need more emphasis on relational concept of equality learning because they usually just understand the operational concept of equality and that is not sufficient for their further algebraic lessons.

### 2) Material deliverance

The material deliverance by teachers usually done in traditional ways. A study done by Rossnan (2006) in context of general teaching of mathematics found that traditionally, teachers teach students memorize mathematical concepts without actually working through problems and comprehending the reason behind the mathematics skill. Gordillo and Godino (2014) stated that most teachers still hold on traditional belief in algebra, that focuses to results, rules, and procedures. Seeing this condition as a phenomenon, a reformation in teaching is needed to help bridging students' thinking from abstract to concrete, from informal to formal, and from experience to concept. The study conducted by McNeil and Alibali (2005a) showed that students in middle school do not tend to see a relational understanding of the equal sign unless they have the contextual support. Knowing that students' interpretations of the equal sign can be influenced by context, therefore it is important for teachers to examine the contexts in which students can see the equal sign. Teachers can conduct a contextual concept teaching of equality using the one and only, Realistic Mathematics Education.

### **The Theory of Realistic Mathematics Education (RME)**

Realistic Mathematics Education (RME) is the answer to the need of worldwide mathematics teaching reformation. In 1971 The Freudenthal Institute (FT), University of Utrecht was established and all set in order to improve the quality of Mathematics teaching in all schools in Netherlands (Dickinson & Hough, 2012). This establishment was the first stepping stone to the development of a pedagogical mathematics theory called Realistic

Mathematics Education (RME). The development of RME was all started by an idea proposed by Hans Freudenthal, a Dutch mathematician, that mathematics should be considered as a human activity (1991). This is a very significant idea of Freudenthal because from this idea, it was then formalized the principles of the RME theory: guided reinvention, didactical phenomenology, and emergent models.

Over time, there have also been many journals, articles, and books reviewing the effectiveness of RME in mathematics teaching and learning. It all shows that the development, trials, and revisions of the theory of RME done by so many educational institutes has been useful and beneficial.

#### 1) The Principles of the Theory of RME

The root of RME is considering mathematics as a human activity. It is very important to remember because that idea helps educator comprehend the RME guide through it's principles. In general term, RME is about teaching and learning mathematics contextually. The context problems themselves are the foundation for progressive mathematization, and through mathematizing, the students develop informal context-specific solution strategies from experientially realistic situations (Gravemeijer & Doorman, 1999). According to Freudenthal, there are three underlying principles of RME: (1) guided reinvention, (2) didactical phenomenology, and (3) emergent models (Freudenthal, 1991).

The first principle, guided reinvention, through progressive mathematization suggests that instructional activities should provide students with experientially realistic situations, and by facilitating informal solution strategies, students should have an opportunity to invent more formal mathematical practices (Freudenthal, 1973). The significance of this principle is that students are enabled to reinvent the "concept" themselves in which the process of "memorizing" is not needed. Therefore in the guidance of the teachers, students have to be facilitated with the chance to undergo the process by which mathematics was invented. Given the activity, students will be able to see the construction of mathematics. Gravemeijer (2008) stated that, "Guidance by teachers and textbooks is not only needed to ensure that the mathematics that the students invent corresponds with conventional mathematics, but also to substantially curtail the invention process" (p. 285).

The second principle is didactical phenomenology. Freudenthal's definition didactical phenomenology is the study of the relation between the phenomena that the mathematical concept represents and the concept itself (1973). The didactical phenomenology suggests ways of identifying possible instructional activities that might support individual activity and whole-class discussions in which the students engage in progressive mathematization (Gravemeijer, 1994). The phenomenology in the context is obviously referring to phenomenology of mathematics. Therefore, this principle focuses on making the students able to interpret a phenomenon mathematically in order to solve the problem.

The third principle is emergent models. This principle is about the activity of modelling in the conception that the modelling may be employed by the students when solving a contextual problem (Gravemeijer, 2008). The modelling might involve drawings, making tables or diagrams that will be the bridge to connect students informal mathematical thinking and the formal mathematics thinking. This will help student develop an understanding of mathematical relations. Then, when the students have experienced similar situations and problems, they can develop more mathematics relations and strategies.

## 2) Christian Perspective Concerning the Theory of RME

There are millions of God's tools to reveal His glory and wisdom and mathematics is one of them. It is Christian Mathematics teachers' obligation to be Truth-driven in establishing mathematics lessons that will make the students know the beauty of Mathematics, the beauty of learning equations, geometry and any other branches of Mathematics. Explaining the connectivity between mathematics and divinity, in the journal of Reformed Perspective Vol. 27, John Byl stated that The Bible clearly states that man was created in the image of God which not only includes the righteousness but also the rationality and creativity that involves the capacity for abstract thought, as well as the ability to reason, to discern and to symbolize (2008, p. 12). Therefore it implies that everyone, all students in the classroom are able to do mathematics regardless of their speed in understanding. It is the teachers' role to teach creatively and compassionately to make the students truly learn mathematics.

The theory of RME is one of many approaches found by scholars in order to increase students' mathematics learning quality at the present time. However when teachers decide to apply a strategy or method or approach or media, teachers must know the exact basis of what is applied. There are things that are aligning the Biblical perspective and there are things that are not. The teachers' job is not to accept one or any philosophy but to gain the insight about the fundamental issues of life and to find the possible answers (Knight, 2009, p. 79).

### **The Theory of RME reviewed from the Christian Perspective**

According to the Oxford Dictionary, the word 'realistic' is defined as representing things in a way that is accurate and true to life. Based on the same dictionary this quality or fact of representing a person or thing in a way that is accurate and true to life is stated as 'realism'. This "ism" was first well-defined by Plato's student, Aristotle. It has a focus on how the physical world is important. Reviewing realism from educational philosophy, according to George K. Knight (2009) in his book "Philosophy and Education", for realists, students are considered as useful organisms, the subjects of the law of nature, that can comprehend the order of nature through the sensing experiences which then force them to deal with "reality" (p. 67). This what is argued by Christians, as stated by Knight,

that the naturalistic assumption proposed by realism leads to deism, agnosticism, and even atheism more than to the personal Christianity itself. Realism minimizes the importance of humanity in order to be objective and scientific. In realism, there is no human free will and the value of humanity. (2009, p.79)

In realism, humans are just a subject of the nature, the comprehender, a learner, a useful organism who has no freedom in choosing. Humans are just a responder of environment stimulus. Knight (2009, p. 67) stated that in realism, students are part of the universal machine to be programmed just the way computers are programmed.. However, Christian teachers will not treat students as machines to gain their mathematics knowledge. Christian teachers facilitate students as the image of God through RME to see the how beautiful mathematics is. Using RME, students have to discover mathematics in reality using their senses given by God to translate their informal understanding into formal mathematics concepts.

At the end of the day, it is important for teachers have that awareness to think of any possible alternatives in order to develop personal philosophy that is not only going to guide

in the process of teaching but also provide the foundation of decision-making in students' daily life (Knight, 2009, p. 29).

## **THE USE OF REALISTIC MATHEMATICS EDUCATION IN TEACHING THE CONCEPT OF EQUALITY**

The key of relational understanding of equality is to comprehend that both sides of the equal sign must be balanced. Any treatment (addition, subtraction, multiplication and division) that is done to one side must be done to the other side as well. Van de Walle, Karp and Williams (2010) clarified that one way to solve an equation is through "balancing". Therefore, to understand the concept of balancing somehow becomes necessary to understand the concept of equality. Teachers can use any media such as balance scale and algebra tiles to apply the RME theory through this concept of "balancing". It has to be remembered to let the students do the observations and undergo the experience of how the balance really works themselves in the guidance of the teachers.

### **The Practical Implementation**

#### 1) Guided reinvention

This principle holds on the believe that students will learn better when they discover the concept themselves. They will be given the situated conditions to experience and see the process of how particular mathematical concepts are invented. According to this principle, the first process includes two components: activity and then the concept discovered. Here, teachers are the 'facilitator' who guides the students to invent, to discover a concept based on their observation.

#### *Example 1. Balance scale*

Teachers are going to let the students use the balance scale with the objects (which can be anything appropriate) by themselves and let them discover their very first concept of "balancing". Teachers have to guide the students with clues and trigger questions like "what do we use this for?", "what do we call this situation when both sides are in the same position?" until the students come up with the concept.

#### *Example 2. Algebra Tiles*

Balance scale is not the only way to help the students understand the concept of balancing. Other than 'weight', there is also 'amount' that can be represented by units to see the 'balance' and algebra tiles is a great tool to start up with. A set of algebra tiles is one of the powerful manipulative for students to explore and express mathematical problems into algebra (Heddens & Speer, 2001). Using algebra tiles, teachers can also represent the negative and positive value in different colors.

#### 2) Didactical Phenomenology

All creation, reveals God. Known theologically as "general revelation", nature, environment, life-phenomenon, science, and also mathematics are given for all human beings to see God's great work. In Romans 1:20 it is written, "For His invisible attributes, namely, His eternal power and divine nature, have been clearly perceived ever since the creation of the world, in the things that have been made. So they are without excuse." From what students observe from their surroundings, students are guided by teachers to see that

the concept they learn can be applied in life. After discovering the situation of 'balancing' they just got to observe, they will have to think mathematically to get into the learning topic. Here is when the mathematization skill is being trained. Here is when the student see the role of equal sign. Here is when the students see clearer the term 'both sides'. Here is when the students first get introduced to the concept they just discovered – equality. Here is when the students contextually see the relational understanding of equality.

### 3) Emergent Models

God created the earth, and men develop the symbols that represent their discovery of the phenomenon embedded in the creation. Miller (2010) stated "God designed an orderly universe that is governed by laws which can be described through the language of mathematics" (p. 5). There are symbols to represent quantities, weight, length, ratio, balance, and etc. Here is when the students have to build their own model. As already exposed in the literature review, the modelling might involve drawings, making tables or diagram that will be the bridge to connect students informal mathematical thinking and the formal mathematics thinking.

#### *Example 1. Drawing*

Students can draw the balance scale that has "equal" sign in the middle.

#### *Example 2. Table*

Students can create their own tables containing the tiles and color their own tiles for an equation.

This principle does not limit the students with particular instruction. They can do their own modelling according to their best preference in learning in teachers' guide. This principle has more roles in lesson application according to the learning objectives. By being able to construct their own model, students are led to see the mathematical concept – equality in many forms and translate their the abstract thinking to a concrete visualization.

### **The Approach**

In implementing the RME theory in teaching mathematics it is important to determine the teaching approach to apply. Basically, there is common integration usually applied in the classroom according to the subject or the topic. There are teacher-centered and student-centered approaches. Valerie Zonnefeld (2015) stated that, "teacher-centered approaches have traditionally been the norm in mathematics classrooms. This approach is characterized as teaching by telling. The educator disseminates knowledge of procedures while the students absorb it" (p. 128). Many educators and researchers have criticized towards this approach because instead of making the students "learning mathematics by doing", the only person learning in a teacher-centered classroom is the teacher (Benson & Findell, 2012). While teacher-centered approach focuses on the "telling", student-centered approach is based on the theory of constructivism that allows the students to create their own knowledge. Valerie Zonnefeld (2015) stated that,

Taken to an extreme, student-centered approaches allow a student to decide that  $2 + 2 = 5$ . This is a dangerous step toward social constructivism in which the

bedrock beliefs of Christianity become irrelevant as students construct their own realities. This is inconsistent with Christian beliefs of absolute truth (p. 128).

Zondefeld's extreme example shows that any statement, any thought, any free ideas can be knowledge in a student-centered classroom. There are no boundaries in reasoning and interpreting. What could be the possible reasons why students think that  $2+2=5$  is true? The answer is anything could be a possible reason. For example, there is an English rock band called Radiohead. The band was formed in 1985 and has released many songs. One of them is titled  $2+2=5$ . One student in the classroom idolizes this band so much that he believes that  $2+2=5$ . That simple extreme example is the reason why a student-centered classroom can not be used in a RME-based learning. Therefore this paper would suggest the combined approach that is called subject – centered in applying the RME theory. In a subject-centered classroom, both the students and the teacher are actively involved, but it is the subject that takes center stage (Zondefeld, 2015, p. 129). This approach is about a connection between the students and the subject which is being connected by the teachers' help. In RME, There is a step "guided reinvention" in which teachers' role is the core of the mathematization process. Speaking of the "guided reinvention" Gravemeijer (2008) stated that, "Guidance by teachers and textbooks is not only needed to ensure that the mathematics that the students invent corresponds with conventional mathematics, but also to substantially curtail the invention process" (p. 285). According to the statement of Gravemeijer teaching using RME has to be done with the teacher's guidance. It is also important for teachers to critically watch over the condition of the teaching and learning (students understanding through conversations, students excitement) process so that teachers can know whether or not the guidance is enough. Meanwhile based on her experience in applying the subject-centered approach in teaching mathematics, Valerie Zondefeld (2015) testified that,

Since that time, I have worked to adjust the focus of my classroom from students to the subject, seeking to lead my students to uncover the truth God has placed in mathematics. God's truth takes center stage. Similar to the student-centered approach, I remain the "guide on the side" and still plan learning experiences that encourage my students to be actively involved in their learning. One of the advantages of guiding students is that when students discover a concept on their own, they internalize it and learn it at a deeper level with greater retention (p. 129).

Subject-centered approach could be beneficial if the teachers know their main roles and necessary competencies in the classroom. Teachers have to engage the students personally and know their needs. Teachers not only teach the lesson but also give students meaningful value to apply in daily life. Zondefeld has testified that teaching mathematics can be done in a biblical way that truth of God in mathematics is the center of the learning process. The center subject of every learning is God's truth, not equation, or algebra, or any topic particularly. All mathematics topics hold together as God's truth, hold together in Jesus Christ who is before all things (Colossians 1:17). In the end, students who are created in the image of God know what they learn, why the need to learn that, and how to respond to creation to glorify God by learning about His creation.



## CONCLUSION

The literature shows that students can be very weak in their understanding of equality concept and that can be very problematic for future learning. However, so many teachers still use traditional methods in teaching mathematics concept which by several studies have been proven not efficient. Teaching reformation in the mathematics classroom is needed and there is the theory of RME that will not only help the students learn but also help the teachers to demonstrate the importance of mathematics for daily life. The use of balancing concept in teaching the concept of equality in subject-centered approach is aligning the principles of the theory of RME and also helps create the learning atmosphere that is effective and efficient for both teachers and students. The three principles of the RME theory is very suitable for the teaching of mathematical concepts without forcing students to memorize. However teachers have to be critical in implementing the theory related to the value contained in it in delivering the lesson. Students are not just a subject of the nature/environment. Students are the image of God who are given the ability to see God's truth and beauty through learning mathematics. This literature review has provided the expositions of how RME theory can be beneficial in teaching the concept of equality. However, the researcher is fully aware that the teaching using this theory can be time-consuming. Therefore this paper suggests to teachers who are willing to teach using the RME theory to precisely learn the arrange the lesson plans, and be wise in determining the topics that need more concern. This paper also suggest teachers to be careful in designing the activities/realistic situations in order to create a maximum result of progressive mathematization according to the three principles of RME theory.

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