

# THE EFFECT OF THE EUCLIDEAN GEOMETRY SHORT LEARNING PROGRAMME USING THE WORKED-OUT EXAMPLES TEACHING APPROACH ON MATHEMATICS TEACHERS' PERFORMANCE

Msebenzi Rabaza<sup>1</sup>, Joleen Hamilton<sup>2</sup>  
<sup>1,2</sup>University of the Free State, SOUTH AFRICA

Correspondence email: [RabazaM@ufs.ac.za](mailto:RabazaM@ufs.ac.za)

## ABSTRACT

The studies on the worked-out examples teaching approach (WOETA) have been investigated globally and in South Africa, showing gains in learners' performance. This study reports on the Euclidean geometry short learning programme (EGSLP), which used worked-out examples teaching approach (WOETA) intervention for secondary mathematics teachers enrolled in one university campus in the Free State province. This study used one group pretest-posttest research design. 65 participants completed the pre-test, participated in WOETA, and later completed the post-test. The WOETA engaged the participants in the Euclidean geometry six topics, straight-line geometry, classification of triangles, Pythagoras, similarity and congruency, properties of polygons, and measurement that helped them to acquire procedural and conceptual knowledge. The findings revealed that the WOETA improved the participants' performance in two topics, straight-line geometry, and the classification of triangles. In contrast, it failed in four topics Pythagoras, similarity and congruency, properties of polygons, and measurement. Cronbach alpha was used to calculate the reliability of the pre-test and was 0,52, while the post-test reliability was 0.30, and these values were deemed not acceptable and not reliable. The findings also revealed that the WOETA does not affect participants' Euclidean geometry performance.

**Keywords:** Euclidean geometry, intervention, performance, scaffolding, worked-out examples teaching approach

## INTRODUCTION

For more than 30 years, the worked-out examples teaching approach has been used worldwide to reduce the cognitive load and improve learners' performance in mathematics. Traditionally worked-out examples teaching approach (WOETA) effectively supports initial cognitive skill acquisition (Renkl et al., 2007). The scaffolding as a form of support to WOETA requires well-organised and structured activities to help the students to follow each step to get to the solutions. Students in this study were secondary mathematics teachers who enrolled in the Euclidean geometry short learning programme (EGSLP) and were also referred to as participants. Kim and Lim (2019) assert that the scaffolding types vary according to the interaction

between the students (mathematics teachers) and tutors (planned, adaptive) sources and examples presented (peer, teacher, technology), and the function (implicit, explicit) or the purpose of using scaffolding (conceptual, cognitive, procedural, strategic). On the latter, procedural scaffolding looks at how the available tools are used, and strategic (intrinsic) scaffolding assists learners in approaching and analyzing learning tasks (Kim and Lim 2019). The mathematics teachers seem to struggle to teach Euclidean geometry in ways that make sense to learners, Manson and Ayres (2021) focused on the completion of the tasks which are partly worked out examples. However, this study focuses on the procedural scaffolding through WOETA to teach mathematics teachers who were not introduced to Euclidean geometry in their secondary education schooling and teacher education institutions (TEI). The explicit scaffolding looks at relieving the working memory of mathematics teachers, which enables them to exploit the inhibition ability (Edelsbrunner & Grimm, 2021). Furthermore, the internalisation of the explicit support is essential for students to make sense of what they learn; therefore, teacher scaffolding is not essential since the students can provide their own support (Van der Pol, Volman, & Beishuizen, 2010) for learning Euclidean geometry activities. Therefore, mathematics teachers require explicit support and scaffolding to do well in Euclidean geometry tests.

The WOETA are presented differently to different professional development programmes within different contexts and follow different scaffolding approaches to minimise the procedural complexities in mathematics solutions. Moreover, there are varying complexities to a solution procedure across mathematics problems and are adopted to meet the principle of complete essence (Liao, 2019). However, the complexity of element interactivity depends on a combination of both the nature of the information and the knowledge of the person processing the information (Sweller et al., 2019). For example, the worked-out example complexity requires that the student's prior knowledge of structure is developed. The student connects worked-out examples to the context to make meaning also plays a critical role.

Learners in South African schools had an option to choose to write Euclidean geometry in the examination paper 3 of the further education and training phase (FET-phase) until the year 2013. The learners' choice of Euclidean geometry in schools led to challenges when they enrolled in universities in mathematics teacher education programmes after completing their school careers. Based on the Curriculum Assessment and Policy Standards (CAPS) for further education and training phase mathematics in South Africa, Euclidean geometry weighs 15% of the content knowledge for grade 12 mathematics. Various topics should be mastered in Euclidean geometry including circles, perpendicular lines through the center, chords, and midpoints, angle at center theorem, cyclic quadrilaterals, tangents, solving circle geometry problems, providing reasons for statements, problems, and proofs using geometric properties (Pearson, n.d.). Without knowing these concepts, mathematics teachers find it a challenge to assist the learners when teaching

these topics. Moreover, Risnawati, Adrian, Azmi, Amir, and Nurdin (2019) agree that one of Euclidean geometry learning goals in schools is to provide a systematic attitude and habits for learners about the relationships between the geometric structures and clarifications between the shapes. Without the necessary background knowledge of geometry, especially Euclidean geometry, mathematics teachers in the FET may struggle to help learners learn and understand.

The mathematics teachers involved in this study were enrolled in EGSLP on one university campus in the Free State province of South Africa. This group was not introduced to the Euclidean geometry teacher education programme in their schooling and teacher education program. The (WOETA) was used to develop their knowledge through scaffolding as an intervention strategy to develop mathematics teachers' knowledge and performance for the 2020 cohort. Scaffolding was used to help the participants acquire procedural knowledge and be assessed on the six Euclidean geometry topics selected in this study. The worked-out examples are teaching and learning materials that do not only consist of the problem but include a step-by-step representation of the problem's solution (Tropper et al., 2015). Our approach is rooted in self-regulation as defined by Meyer and Turner (2002) looked at from a traditionally influenced by environmental factors and viewed as a social process that implicitly binds the individuals and their social context.

The WOETA fosters scaffolding by enhancing the teacher's presence and independent learning facilitated through scaffolding (Khatri, 2021). However, many students are daunted by learning the formal logic and deductive reasoning necessary elements for Euclidean geometry (Ubah, & Bansilal, 2019). Moreover, Jones (2002) asserts that Euclidean geometry is full of surprising theorems and interesting tasks, is open to different teaching and learning approaches, and is intimately connected to cognitive mathematical development. Some studies in mathematics that investigated the WOETA include Özcan (2017), who investigated the relative impact of using four different types of worked-out examples: worked-out examples with self-explanation prompts, fading out examples without self-explanation prompts, and fading out examples with self-explanation prompts. The results showed that using self-explanation prompts in combination with backward fading worked-out examples fosters learning in transfer and follow-up studies. This suggests that the WOETA with self-explanation prompts, and backward fading needs to include representations to help mathematics teachers to understand Euclidean geometry.

## **LITERATURE REVIEW**

Euclidean geometry proves to be one of the most challenging topics to teach and learn in mathematics (Shongwe, 2019). Moreover, the most persistent proof-related difficulties identified among learners in secondary schools and universities are a consequence of the confusion about the functions of proof in mathematics (Shongwe, 2019). Furthermore,

mathematical proof performs various mathematics functions, including verification, explanation, communication, discovery, systematisation, and intellectual challenge (Shongwe 2020). These functions will be complicated for mathematics teachers to display if they lack Euclidean geometry knowledge since they need verification and proof for correctness. Therefore, mathematics teachers may struggle to introduce Euclidean geometry reasoning and proof if they lack the knowledge. Nevertheless, a narrative can help provide opportunities for teachers to introduce reasoning and proof to students (Thompson et al., 2012). The Euclidean geometry tasks play an essential role in assisting learners to reason, and mathematics teachers observe the learners' reasoning through their responses to activities. The tasks allow teachers to engage students in practice with reasoning and proof (Thompson et al., 2012). Mathematics teachers who avoid the Euclidean geometry teaching and learning activities in class may not be able to help learners to show their reasoning and proof in class.

### **Challenges in Teaching Euclidean Geometry**

Teaching Euclidean geometry in secondary schools can be challenging to mathematics teachers without the concept's prior knowledge. Nevertheless, mathematics teachers in secondary schools are expected to teach Euclidean geometry to help learners make sense of the concept. Several researchers have investigated the teaching of Euclidean geometry for years (Usiskin, 1972; Nikoloudakis & Dimakos, 2014; Sibiya, 2020). What is worth mentioning is that, currently, the impression is that learners' poor performance might be influenced strongly by the teaching methods (Sibiya, 2020). The mathematics teachers need to note that the content knowledge drilled in the learners' minds is a phone swipe away. Still, the ability to make sense of information (Haber, 2020), especially Euclidean geometry, requires mathematics teachers to provide approaches and strategies with activities to develop logical and creative reasoning. To improve logical reasoning, mathematics teachers need to allow learners to consider words rather than reducing them into symbols to fit into a structure (Haber, 2020).

Some teachers struggle to teach Euclidian geometry for learners' understanding. Hence, Ubah and Bansila (2019) state that Euclidean geometry's understanding requires fluency in moving between the visual representation using geometric figures and the symbolic representations that use symbolic notations for congruency and similarity. Therefore, teaching Euclidean geometry without representations becomes difficult for both the teachers and learners. Moreover, learners struggle to answer items without diagrammatic representations, and teachers need to emphasise the role played by visualisation in the teaching and learning of mathematics (Ngirish & Bansilal, 2019).

## **The Worked-Out Example Teaching Approach to Improve Students' Performance in Euclidean Geometry**

Many studies claim that the WOETA improves learners' performance (Renkl, Hilbert, Schworm, & Reiss (2007), Khatri, (2021), Zormelo (2018)). These authors claim that WOETA improves learners' performance in Euclidean geometry. There are limited studies that investigate the WOETA and mathematics teachers' performance. The learners' studies were used to understand how WOETA improves the performance of learners. After that, understand the teacher's performance in Euclidean geometry when they are introduced to WOETA. According to Zormelo (2018), learners introduced to WOETA performed stronger academically in Euclidean geometry than learners who used the conventional teaching approach. In addition, the study also found a statistically significant difference in performance between the two groups, and the WOETA claimed the improvement of learners' performance in Euclidean geometry. In contrast, Baars et al. (2013) assert that completed worked-out examples resulted in learners underestimating their future test performance. Whereas partially worked-out examples made mathematics teachers less confident about future performance than studying worked-out examples fully. This suggests that fully worked-out examples may improve mathematics teachers' Euclidean geometry performance in mathematics.

### **Conceptual Framework**

The study used Van Hiele's five levels of geometric understanding as a theoretical framework. The five levels include visualisation, analysis, abstraction, deduction, and rigor (Mason, 2002). Firstly, visualisation - at this level, mathematics teachers can recognise geometric figures by their shape, compare them with their prototypes or everyday things, and categorise them (Vojkuvkova, 2012). The visualization technique enables mathematics teachers to understand how to solve the problem by understanding the concept and meaning rather than applying memorised rules and procedures (Bicer et al., 2022). The WOETA may help mathematics teachers memorize the steps to the problem solution using scaffolding. Secondly, analysis – Mathematics teachers at this level can describe a shape's properties and understand that shapes in a group have the same properties. At this level, an analysis of geometric concepts begins through observation and experimentation through students, and they also begin to discern the characteristics of figures (Crowley, 1987). For mathematics teachers, scaffolding examples may help them make sense of and describe the properties of Euclidean geometry shapes. Thirdly, abstraction – mathematics teachers can create meaningful definitions and give informal arguments to justify their reasoning (Mason, 2002). The mathematics teachers need to understand and avoid abstraction in the Euclidean geometry concept by reducing it and providing scaffolding through various representations to help create a mental image. Fourth, deduction - has two kinds, formal and informal. With the informal deduction, mathematics teachers can

comprehend the relationship between shapes and create relationships (Abdullah & Zakaria, 2013).

With the formal deduction, learners at this level appreciate the meaning and importance of deduction and the role of postulates, theorems, and proofs (Abdullah & Zakaria, 2013). For this study, the informal deduction will help mathematics teachers try a variety of worked-out tasks to emphasise the Euclidean geometry concept to learners. The last level of Van Hiele focuses on rigor, and learners understand the way mathematics systems are established (Vojkuvkova, 2012). The five levels of geometric understanding are relevant to the Euclidean geometry taught in South African schools. In addition, Van Putten, Howie, & Stols (2010) state that the question types and the construction of proofs in South African grade 12 paper 3 question paper expect learners to be at level 3 of Van Hiele's five levels of geometric understanding. The five steps were deemed relevant to mathematics teachers helping secondary school learners learn Euclidean geometry.

### **Research Questions**

- What effect does a Euclidean geometry short learning programme using worked-out examples teaching approach to have on mathematics teachers' performance?

### **METHODOLOGY**

This interpretive research study followed a quantitative research approach to select one group pre-test post-test research design to evaluate the effectiveness of the WOETA on participants' performance. The pretest-posttest design was used for EGSLP participants through a pre-test, then administer the WOETA through the EGSLP unit 1, unit 2, and unit 6. After completing the unit's post-test data were collected using the same measure (Bell, 2010). The study collected data from a total number of 65 participants who completed the pre-test and post-test using google forms. The ethical considerations were observed through UFS-HSD2021/0294/21. The self-designed pre-test and post-test were sent to teachers and experts in the university who were not part of this study. Their responses helped the study with 40 multiple-choice items categorised into six topics, namely straight-line geometry, classification of triangles, Pythagoras, similarity and congruency, properties of polygons, and measurement. Cronbach alpha was used to calculate the reliability of the pre-test and was 0,52, while the post-test reliability was 0.30, and these values were deemed not acceptable and not reliable. The data from the Google forms were analysed using an Excel spreadsheet and presented descriptively and inferentially below.

### **Pre-test**

A link to the Google form app was sent to all the participants using the WhatsApp platform to complete the pre-test. Before completing the pre-test, the participants had to consent by clicking on the button provided on the page and the timer stopped counting after 60 minutes and the test closed. The participants completed the pre-test by choosing the correct answer from the four options given. Considering the participants' workload and the possibility of electricity outage in the form of load shedding in the country, the participants were allowed two days to complete each pre-test and the post-test. However, the participants only had one attempt in answering the pre-and post-test. The length of time may have influenced the results, but we deemed the time essential to accommodate the mathematics teachers to complete the test. One of the challenges of using a Google form to complete the pre-test is that teachers can work together when answering the questions.

### **Intervention**

The EGSLP followed the WOETA to teach Euclidean geometry to secondary school teachers. The intervention approach was categorised into content and reflection activities. This study focused on content activities which are discussed below.

The EGSLP used the WOETA to develop the content knowledge of Euclidean geometry for secondary school mathematics teachers. The WOETA was conducted a week after the completion of the pre-test and lasted for six weeks period. Each participant in the study received a pack of documents, a module guide with worked-out examples, and activity books. The study was conducted during the lockdown, and there were no face-to-face sessions with the participants due to Covid 19 regulations. The participants received a schedule with specific dates on which the different assessments needed to be submitted. The researchers supported the participants through a WhatsApp group. Regular reminders were sent about the progress of the schedule and submissions of the assessments. Participants had the opportunity to communicate with the researchers via WhatsApp to ask for assistance at any time.

The participants worked at their own pace with the help of WOETA on the module guide with units 1, 2, and 6 to complete the six Euclidean geometry topics: straight-line geometry, classification of triangles, Pythagoras, similarity and congruency, properties of polygons, and measurement within the six weeks period. In this study, the WOETA was intended to guide the participants, who have limited Euclidean geometry knowledge, leading to the solution developed by experts in secondary mathematics from one university campus in the Free State Province.

## Post-test

After 65 participants completed the content activities during the intervention using WOETA, they received a Google form link to complete the post-test. The participants clicked on the consent button before they completed the post-test. All the questions in the post-test were like those in the pre-test.

## Data Analysis

Data of participants' pre-test and post-test scores on WOETA in the year 2020 in one university campus in the Free State province were compared. The data analysis techniques include the descriptive data in the form of sample means and standard deviations. The inferential data involve the t-test to compare a paired sample with a level of significance of  $P < 0,05$  with overall data analysis performed using Excel software version 2019.

## RESULTS AND DISCUSSION

The presentation of the results combined the descriptive and inferential data and complemented with the relevant and contrasting literature reviewed relevant to this study's research questions. The table below includes the six topics that were compared on the pre-test and post-test mean scores and the significant difference between each topic. Each topic is discussed below.

**Straight line geometry** – The table shows that the pre-test scores were slightly higher than the post-test scores on the straight-line geometry concept in an EGSLP. Data from the pre-test shows a low spread of scores towards the mean, whereas data from the post-test show a high spread of scores towards the mean. The low post-test mean score from the pre-test mean score suggests that the WOETA examples had no impact on the participants' performance in straight line geometry. Table 1 shows a statistically significant difference in participants' performance in straight line geometry the pre-test and the post-test mean scores with a P-value of 0,007633, less than  $P > 0,05$ . This suggests that the WOETA influenced the participants' performance in straight line geometry. The finding of this study agrees with the finding by Zormelo (2018), who found that the WOETA showed a statistically significant difference in the performance of learners who received the instruction compared to the control group. The limitation of our study is that there was no control group, and the teachers were working at their own pace in their own places. The find shows an improvement in mathematics teacher performance on the topic straight line, and there is an effect of EGSLP on the performance of mathematics teachers. Substantively, this suggests that mathematics teachers can improve their learner's knowledge of the topic of straight-line in Euclidean geometry.



**Table 1.** Euclidean geometry content domains on pre-test and post-test scores

Euclidean geometry topics	Pre-test mean N = 65	Post-test mean N = 65
<b>Straight line geometry</b>	16,67273	16,2
Standard deviation	0,639865	1,282359
P-value	0,007633	
<b>Classification of triangles</b>	2,709091	2,890909
Standard deviation	0,49713	0,314627
P-value	0,008494	
<b>Pythagoras</b>	2	2,963636
Standard deviation	0	0,26968
P-value	<b>6,29304954170632</b>	
<b>Congruency and similarity</b>	1,563636	1,563636
Standard deviation	0,713978	1,253413
P-value	<b>3,55128563976374</b>	
<b>Properties of triangles</b>	8,854545	8,81818
Standard deviation	0,447966	0,56735
P-value	0,242273	
<b>Measurement</b>	8,709091	8,636364
Standard deviation	0,566756	1,176367
P-value	0,338634	
<b>Average score of all content areas</b>	40,50909091	42,32727273
P-value	0,000019264979	

**Classification of triangles** - Data in table 1 shows that the post-test scores were higher than the pre-test scores after the participants were introduced to WOETA on the topic of classification of triangles. İltüzer and Çevik (2021) state that worked examples were used for more than 30 years as an effective learning and performance improvement strategy. This suggests that the WOETA teaching Euclidean geometry helped the participants to improve their performance in the classification of triangles. Data shows a high spread of scores towards the mean on the pre-test and a low spread of scores towards the mean for the post-test. Data show a statistically significant difference in participants' performance between the pre-test and post-test scores in the classification of triangles, with P-value being 0,008494, which is below the range  $P \geq 0,05$ . This finding shows an improvement in mathematics teachers' performance on the topic classification of triangles, and there is no effect of EGSLP on the performance of mathematics teachers.

**Pythagoras** – Table 1 data shows that the participants scored higher on a post-test compared to the pre-test on the topic, Pythagoras, after participating in the WOETA. This suggests that the WOETA helped the participants to improve their performance on the topic, Pythagoras. Data shows no statistically significant difference in participants' performance in Pythagoras with a P-

value of 6,29304954170632, thus below the range  $P \leq 0,05$ . The finding shows an improvement in mathematics teachers' performance on the topic Pythagoras, and there is no effect of EGSLP on the improvement of mathematics teachers' performance. Therefore, this suggests that other factors may contribute to the improvement, or the improvement may have happened by chance.

**Congruency and similarity** – Data show the pre-test and post-test scores had the same value. This suggests that there was no improvement in performance between the pre-test and post-test. However, some studies disagree with this finding, though they agree that WOETA improves performance (İltüzer & Çevik, 2021; Van Gog et al., 2011). However, data shows no statistically significant performance between the pre-test and post-test scores with a P-value of 3,55128563976374, more than  $P < 0,05$ . Mathematics teachers' performance at the same level shows no improvement in the scores. There is no effect of EGSLP and the mathematics teachers' performance on congruency and similarity topics.

**Properties of triangles – Properties of triangles** – Data shows a higher pre-test score compared to the post-test score. The post-test has a higher spread of scores towards the mean, whereas the pre-test has a low spread of scores towards the mean. This study's finding contrasts with Alreshidi's (2021) finding that students who were taught using WOETA performed better than students who were taught using conventional methods. Therefore, this study's finding suggests that there could be some limitations on the WOETA provided to mathematics teachers. The use of WOETA with a small sample of participants has been the limitation of this study and other studies. However, Tropper et al. (2015) further state that varying contexts would have to be investigated to determine whether the findings represent rather stable patterns of behaviors or if they are strongly dependent on the materials we used. Other factors could contribute to the drop in performance of the post-test scores. Data shows no statistically significant difference in performance between the pre-test and post-test with a P-value of 0,242273, more than  $P < 0,05$ . This finding suggests that the drop in mathematics teachers' performance has no effect of EGSLP on mathematics teachers' performance. Therefore, this suggests that mathematics teachers will continue to struggle to help learners perform better on the topic properties of triangles in Euclidean geometry.

**Measurement** – Data shows higher pre-test and low post-test scores. Data shows a higher spread of post-test scores towards the mean and a low spread of pre-test scores towards the mean. This suggests that the EGSLP explanation of the measurement concept may have contrasted with their prior knowledge of this concept. Data shows no statistically significant difference in performance between the pre-test and post-test scores with a P-value of 0,338634 more than  $P < 0,05$ . This finding suggests the mathematics teacher's performance drop after the post-test, and there is no effect of EGSLP on the drop of mathematics teachers' Euclidean geometry performance.

**The average score for all the topics** – Data shows a lower pre-test score compared to the post-test score. When combining the mean scores of the six topics investigated in this study, data show a statistically significant difference in performance between the pre-test and the post-test scores with a P-value of 0,000019264979,  $P < 0,05$ . The finding suggests that the EGSLP improved mathematics teachers' Euclidean geometry performance, and there is an effect of EGSLP on the improvement of mathematics teachers' performance. Therefore, the improvement suggests that mathematics teachers can transfer Euclidean geometry knowledge to their learners in schools.

## Discussion

The study results have been presented in the previous section, and the authors take some findings based on the research questions. This study sought to determine the effect of the Euclidean geometry short learning programme using WOETA on mathematics teachers' performance. The major finding revealed that the EGSLP using the WOETA showed no statistically significant difference in Euclidean geometry performance. There are limited studies that used the WOETA to improve the performance of mathematics teachers in Euclidean geometry. Some studies, Shongwe (2019), Sibiya (2020), and Tachie (2020), focused on the challenges experienced by Euclidean geometry mathematics teachers and the need to improve their teaching in secondary schools. This study continued from the previous studies to provide an intervention to enhance the teachers' teaching of Euclidean geometry in secondary schools. The findings of this study are categorised into six Euclidean geometry topics, straight-line geometry, classification of triangles, Pythagoras, congruency and similarity, properties of triangles, measurement, and the overall performance of all topics. The findings show that the student's average performance in six topics and the overall performance of the six topics were not consistent. The findings led to realisations regarding the topics' performance before and after the WOETA was completed. Firstly, the performance on the topic, straight-line geometry, dropped. Secondly, the performance on the topic, classification of triangles, improved. Thirdly, the performance on the topic, Pythagoras, improved. Fourthly, there was a tie between the topic's performance, congruency, and similarity. Fifthly, there was a drop in performance on the properties of triangles. Sixthly, there was a drop in performance on the topic, of measurement. Lastly, there was an improvement in the overall performance of the six topics. The drop and improvement in the performance of the six topics and the overall performance of the six topics may be attributed to WOETA EGSLP.

On the one hand, the results showed a statistically significant difference in performance in two topics, straight-line geometry and classification of triangles, and in the overall performance of the six topics investigated in this study. This suggests that these topics weighed more marks than the four topics, and the EGSLP influenced the two topics, including the overall topics' performance. Therefore, it was essential for mathematics teachers who participated in

WOETA to possess more knowledge on the two topics to perform well than in the other four topics. On the other, the results showed no statistically significant difference in performance in four topics, Pythagoras, congruency and similarity, properties of triangles, and measurement. This suggests that the student's performance on the four topics above is attributed to the design of the EGSLP.

### **Limitation of the Study**

The limitations of this study include the use of the smaller group and the lack of the control group. A group of mathematics teachers was used to test the effectiveness of the EGSLP. The significance of the performance in the pre-test and post-test was the major aspect of evaluating the SLP to improve in the next group. This study did not use the control group but compared the same group of mathematics teachers who participated in the WOETA for different topics. There was no face-to-face interaction between the mathematics teachers and the facilitators, but WhatsApp messages and emails were used to communicate with mathematics teachers.

### **The implication of the study**

The findings of the study were not as positive as what was expected. This asks the developers of EGSLP to reflect on the method used in the explanation of the content and adjust where needed. The mathematics teachers can become aware of their performance when working with the WOETA involving Euclidean geometry tasks and they know the steps leading to the solution. The knowledge can help teachers explain the Euclidean geometry steps leading to the solution when teaching Euclidean geometry to learners in secondary schools. This study addresses partially the need to provide alternatives in explaining the Euclidean geometry steps to get to the solution. It presents a foundation for more specific activities involving procedural knowledge in the learning and teaching of Euclidean geometry.

### **CONCLUSION**

This study discussed the performance of mathematics teachers who were introduced to EGSLP intervention which used the WOETA to develop mathematics teachers' knowledge of Euclidean geometry. This study aimed to find the effect of Euclidean geometry short learning programme using worked-out examples teaching approach on mathematics teachers' performance. It was found that the EGSLP intervention which used the WOETA had no effect on the performance of the mathematics teachers. Therefore, the EGSLP that used the WOETA is better suited for supporting mathematics teachers on straight-line geometry and the classification of triangles. Nevertheless, mathematics teachers need to experience less abstraction in the Euclidean geometry activities to comprehend and explain the propositions of given activities. Therefore, the EGSLP should be presented meaningfully and efficiently to

mathematics teachers by ensuring constant online learning to provide scaffolding and minimize the abstraction of the concepts.

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