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LEARNING TRAJECTORY OF QUADRATIC INEQUALITY

Kimura Patar Tamba[1], Melda Jaya Saragih[2], Tanti Listiani[3]
[1,2,3)]Universitas Pelita Harapan, Tangerang, Banten

Correspondence email: kimura.tamba@uph.edu

ABSTRACT

A learning trajectory offers a description of key aspects in planning mathematics learning. It also helps teachers follow and interpret students' mathematical thinking, so that learning can be developed in accordance with the characteristics of students, and even become a tool for teachers to develop curriculum. There are three main components of learning trajectory: learning goals, learning activities, and hypothetical learning processes. In this article, we constructed a learning trajectory of the quadratic inequality. This qualitative study used didactical design research with 105 grade 10 students as the participants. In the prospective analysis step, didactic design, learning obstacle, and quadratic inequality system were analyzed. Based on the results of this analysis, we constructed hypothetical learning trajectories in the form of didactical design. Then, hyphothetical learning trajectories were implemented in the learning process. Student's responses were analyzed qualitatively. Results of this analysis were used to revise the learning trajectory in order to obtain alternative trajectory learning outcomes of theoretical and empirical analysis. Finally, this article offers an alternative learning trajectory of quadratic inequalities that are different from the existing learning trajectories presented in the current textbook. The learning trajectory that is offered is the learning quadratic inequality which starts from the function approach.

Keywords: learning trajectory, quadratic inequality, didactical design research, learning obstacle

ABSTRAK

Learning trajectory (LT) menawarkan sebuah deskripsi akan aspek kunci dalam perencanaan pembelajaran matematika. LT juga membantu guru belajar dalam mengikuti dan menginterpretasi cara berpikir matematisnya siswa, sehingga pembelajaran dapat dikembangkan sesuai dengan karateristik siswa, bahkan menjadi alat bagi guru untuk mengembangkan kurikulum. Ada tiga komponen utama dari learning trajectory, yaitu: tujuan pembelajaran (learning goals), kegiatan pembelajaran (learning activities) dan hipotesis proses belajar siswa (hypothetical learning process). Dalam artikel ini akan dikonstruksi sebuah LT pertidaksamaan kuadrat. Penelitian ini menggunakan pendekatan kualitatif dengan didactical design research. Adapun partisipan sebanyak 105 siswa kelas X. Pada awal penelitian ini, dilakukan analisis propektif yaitu analisis atas materi pertidaksamaan kuadrat, hambatan belajar dan tingkat berpikir siswa. Kemudian dari hasil analisis ini disusunlah Hipotetical Learning Trajectories yang berupa desain didaktis. Desain didaktis berdasarkan Hypotetical Learning Trajectories ini diimplementasikan dalam pembelajaran. Respon siswa dianalisis secara kualitatif. Hasil analisis ini digunakan untuk merevisi Learning Trajectory, sehingga diperoleh Learning Trajectory alternatif hasil analisis teoritik dan empirik. Akhirnya, artikel ini menawarkan sebuah alternatif learning trajectory pertidaksamaan kuadrat yang berbeda dengan learning trajectories yang ada pada buku pelajaran sekarang. Learning trajectory yang ditawarkan adalah pembelajaran pertidaksamaan yang dimulai dengan pendekatan fungsi.

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Kata Kunci: lintasan belajar, pertidaksamaan kuadrat, *didactical design research*, hambatan belajar

INTRODUCTION

Many researches have shown that Indonesian students' mathematical thinking is low. A research (TIMSS) done by Mullis et al. (2016) confirmed the low mathematical thinking among Indonesian students. It showed that only 4% students who have good mathematical thinking, for instance in reasoning skill (high order thinking). The low mathematical thinking among Indonesian students is caused by the obstacle in learning specifically the quadratic inequality.

Inequality is one of the crucial topics in understanding various topics in mathematics such as algebra, trigonometry, and analytic geometry (Tsamir & Almog, 2001; Bazzini & Tsamir, 2001; Bicer, Capraro, & Capraro, 2014). Therefore, in designing the learning system which is proper to students' learning trajectories require an analysis of the developmental progression and student's conceptual thinking on the quadratic inequality material. However, the existing didactical design in the classroom, tends not to perform the analysis on the thinking development and student's conceptual on the material. It is shown that in all the didactical design used by teachers there is no difference in learning trajectory. While on Tamba's research (2015) showed that students found some obstacles during the use of quadratic inequality through existing didactical design. Thus, it is important to do an empirical analysis to find out alternatives in the design of didactic learning trajectories quadratic inequality.

It is one of a Christian teacher's responsibilities to thoroughly construct a learning design (didactic design) that is relevant to students' uniqueness which is their learning trajectories. Van Brummelen (2009) argued that a teacher must arrange a learning design based on the continual analysis and reflection of their teaching experience.

LITERATURE REVIEW

In preparing the didactic design, teachers should consider about how students will go through the learning trajectory so the learning objectives can be achieved. Having these considerations, teacher is able to design the didactic situation in accordance with the students' learning trajectories. Simon (1995) used the hypothetical learning trajectories term first to show how teacher designs a learning.

Simon used the word "hypothetical" to indicate that part of the learning trajectories is flexible, where teacher can change the learning objectives and adapt the planning aspects based on the teacher's perception on students understanding levels and teacher's observation on performance of students while doing the tasks in the classroom. Therefore, the actual learning trajectories aren't known before. According to Simon (1995), there are three main components of the learning trajectories: learning goals, learning activities and the hypothetical learning process. These are developed by Clements and Sarama (2004) into:

(1) learning goals; (2) developmental progression of thinking and learning, and (3) sequences of instructional tasks.

Constructing a didactical design in accordance with Hypothetical Learning Trajectories (HLT) is set by identifying the learning objectives for students—first component. Then, the didactical design is designed based on the learning objective. HLT design is based on the teacher's knowledge of mathematics, teacher's knowledge of mathematics activities and representations, teacher's hypothesis of student's knowledge, teacher's theories about mathematics learning and teaching, and teacher's knowledge of student's learning of particular content.

To construct the learning trajectories, Simon and Tzur (2004) gave four principles that must be considered:

1. HLT is designed based on the students' current mathematics knowledge.
2. HLT is a tool use for planning particular mathematical contents.
3. The exercises or the worksheets as tools to promote the learning on mathematics content are the key of the teaching process
4. Teachers must modify the HLT aspects if it's not based on the students learning process continuously.

Therefore, to have learning trajectories that are match with students' learning process, Hypothetical Learning Trajectories (HLT) must be implemented and revised according to students's response in the classroom.

RESEARCH METHODOLOGY

This qualitative study used didactical design research. There are three steps in conducting this research (Suryadi, 2013) that are (1) situation didactical analysis before learning process in the form of a didactical design hypothesis (prospective analysis); (2) Metapedadidactical analysis, and (3) Retrospective analysis which relating the outcome of situation didactical analysis hypothesis and metapedadidactical analysis. In the prospective analysis step, current didactic design, learning obstacle, and quadratic inequality system were analyzed. This analysis was used to design hyphotetical learning trajectories. Then, in metapedidaktik stage the didactic design arranged from Hyphothetical Learning Trajectories was implemented in the learning process. Throughout this implementation process, all students' responses to instructional taks were observed and interview about their difficulty. Finally, in prospective analysis stage, hyphotetical learning trajectories was modified based on the prior metapedadidaktik analysis. This study was conducted at ABC Senior High School, Bandung, and the participants of this research were 105 students of grade 10.

RESULT AND DISCUSSION

Hypothetical Learning Trajectories

The arrangement of Hypothetical Learning Trajectories is made in prosepective analysis stage through analyzing learning obstacle, current didactic design (textbook, lesson plan, curriculum), learning obstacle and quadratic inequality system (in mathematics' context). According to this analysis result, hypothetical learning trajectories will be constructed. There are three components that must be considered in constructing a hypothetical learning trajectories: (1) learning goals; (2) developmental progression of thinking and learning; and (3) sequences of instructional tasks (the learning activity).

1. Learning Goals

Based on the mathematical knowledge of the students who have studied the quadratic equations, linear inequality and the learning objectives on the curriculum of senior high school (Kementrian Pendidikan dan Kebudayaan Republik Indonesia, 2013) then the purpose of the learning quadratic inequality are:

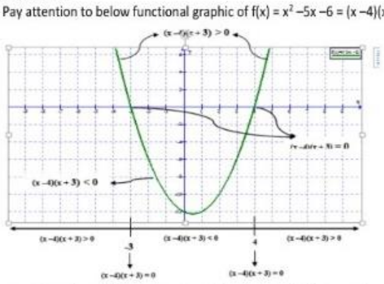
- Students are able to use the properties and rules about quadratic inequality
- Students are able to perform algebraic manipulations in the calculations associated with the quadratic inequality
- Students are able to design and to complete mathematical models of the problems related to quadratic inequality.

2. Developmental progression of thinking and learning and sequences of instructional tasks.

The arrangement of developmental progression of thinking and learning and sequences of instructional tasks is based on current didactic analysis (either used by teacher or research result and expert's judgement), learning obstacle and mathematical knowledge about quadratic inequality.

From an analysis of textbook, lesson plan and students' handout used by a teacher, several findings have been found as follow: (1) A teacher focuses solely on the number line approach and procedur of teaching quadratic inequality; (2) Quadratic inequality which is delivered by a teacher focuses only on manipulating algebra and the number line method; (3) The learning does not stress the difference between equation and inequality (no transition from equation and inequality). This approach causes an obstacle towards students' learning.

Some obstacles faced by students caused by this approach are (1) Students face difficulity in dealing with quadratic inequality when the problem is presented in function or graphical representation; (2) a single approach (sign-chart method) that the students use cannot solve all quadratic inequality problems, and it only makes students undertand it procedurally (Tamba, 2015). The issue of learning obstacle is discussed in a research conducted by Tamba with the same students as a research subject in this study. Learning obstacles in this quadratic inequality system are (1) equation generalization towards inequality (students did not change the inequality symbol when multiplying quadratic inequality with negative number); (2) a generalization of two variables inequality to quadratic inequality (students wrongly understand the relation between graphical representation and quadratic inequality solution).

<table>
<tr><th>Developmental progression</th><th>Sequences of instructional tasks</th></tr>
<tr><td>Quadratic Equation</td><td>Situation-1
A shareholder wants to build a shop on a square-shaped land. That share holder sets aside 2 and 3 metres long land located beside and in front of the shop's yard.
1. What is the area of the land needed by the share holder if he/she plans to have a building area of 72 m^2?
2. That shareholder changes his/her mind because he/she limits the area for not exceeding 72 m^2. What is the area that he/she must have?</td></tr>
<tr><td>Quadratic function and quadratic function graphic</td><td>3. Describe and illustrate the graphical function that represents the area of the house to build?
4. How to decide the size (length of the side) of the land that is needed so that the share holder could build a house as he/she wants?
5. What is the relation between function and graphical function in determining the land size (area?) which the share holder must have?</td></tr>
<tr><td colspan="2">On Situation-1
This situation is structured so that students can recall and use the previous understanding of quadratic equations to solve a quadratic inequality problems. According to Kieran (2004) one of the activities in the algebra is a generalization. Therefore, he thinks that it is important to connect the meaning of quadratic equations in solving a quadratic inequality. This also corresponds to the meaning of the study according to the Brousseau (1997) where students learn if they could use their knowledge to solve the problem at hand. This situation also to anticipate epistemological obstacles that is generalizing the equation to inequality (students were brought in to see the difference with the equation and see that the concept of limited equation to understand inequality).</td></tr>
<tr><td>Quadratic inequality with real number solution x ($\forall x \in A, A \subset R$)</td><td>Situation-2
1. A share holder would like to build a house on a square-shaped land. He/she sets aside 2 and 1 metres long land from the house to build. What is the area of the square-shaped land needed if he/she wants to have a shop area less than 56 m^2?
2. Determine the set of solution of the value x that fulfills following quadratic inequality:
a. $x^2 - 6x + 8 < 0$
b. $x^2 + 5x - 6 \leq 0$</td></tr>
<tr><td colspan="2">On Situation-2
The situation was developed as a continuation of the situation-1 where students are encouraged to use a function approach to determine the solving of a quadratic inequalities problems. Through this situation students are expected to complete inequalities shaped ax2 + bx + c > 0 and ax2 + bx + c ≥ 0 with the function approach f(x) > 0 or f(x) ≥ 0 (graphical). With this situation, students are expected to also see what the meaning of sign "<" and ">".</td></tr>
<tr><td>Quadratic inequality with real number solution x ($\forall x \in R$)</td><td>Situation-3
Do you still remember the situation faced by the shareholder who wants to build a shop? He wanted to build a shop on a square-shaped land by setting aside his/her 2 and 3 metres long yard in the left and in front. However, that share holder changes his mind because he/she wants to buy land in the right side with the same size as the previous land and 70 m^2 land at the back. On the land which he/she buys, there will be a shop with the same area as the land.
What is the required size area of the earlier land in order to build the shop (the previous and the additional one) bigger than 72 m^2?
1. State the shop area in algebra equation.
2. State algebra (function) which denotes that the shop area is bigger than 72 m2.
3. Draw a functional graphic from number 2.
4. Determine all possible values of the length of side of the land which the share holder must have.
5. How is functional graphic in number 3 determines the land size that the share holder must have.
6. Based on question number 1 to 5, determine the value x that fullfils following equation.
a. $x^2 + 4x + 5 > 0$
b. $x^2 - 3x + 7 > 0$</td></tr>
<tr><td colspan="2">On Situation-3
This situation is structured so that students can complete a quadratic inequalities with all real numbers as a solution. This situation is arranged as the continuation of situation-2. This is done so that students could comprehensively understand how to solve quadratic inequalities by using function and graphical approach. With these situations students are expected to see that an empty set and all real number could be the solution for quadratic inequalities.</td></tr>
<tr><td>Quadratic inequality with no solution (solution x, $x \notin R$)</td><td>Situation-4
In situation one, a share holder wants to have a shop which is bigger than 72 m^2, and then he/she want to have a smaller area than that area.
Initially, the shareholder wants to build a shop on a square-shaped land. That share holder sets aside 2 and 3 metres long land located beside and in front of the shop's yard. However, he/she changes the plan beause he/she got additional fund. He/she wants to buy land in the right side with the same size as the previous land and 70 m^2 land at the back. On that land will be built a shop with the same area as the land.
What is the initial area that should be bought so that the whole area of the shop (initial and additional) is smaller than 72 m^2?
1. State the shop area in algebra equation.
2. State the algebra function which denotes that the shop area is bigger than 72 m^2.
3. Draw a functional graphic from number 2.
4. Determine all possible values of the length of side of the land which the share holder must have.
5. How is functional graphic in number-3 determines the land size that the share holder must have?</td></tr>
<tr><td colspan="2">On Situation-4
This situation is structured so that students can complete a quadratic inequalities with solusion $x \notin R$ or in other words the quadratic inequalities has no solutions. This situation is arranged as a continuation of situation-3. This is done so that students could comprehensively understand how to solve quadratic inequalities by using function and graphical approach. With these situations students are expected to see that an empty set could be the solution for quadratic inequalities.</td></tr>
<tr><td>Solving quadratic inequality by Sign Chart method</td><td>Situation-5
Pay attention to below functional graphic of $f(x) = x^2 - 5x - 6 = (x - 4)(x + 3)$:


1. From above illustration, what is the relation of the line number to the graphic?
2. How to determine a set of solution of the quadratic inequality without seeing the graphic (use the number line only)?
3. Arrange the steps to solve quadratic inequality without with (?) the line number only.</td></tr>
<tr><td colspan="2">On Situation-5
This situation is structured so that students can see the solution of a quadratic inequalities with the approaches/methods other than the function and graphical approach. Through this situation students are directed to find a strategy without using the graph of a function, i.e., using the method of sign chart method. With this situation, students are directed to see that on sign chart methods students should see the makers of zero (the value of x to f(x) = 0). According to Bazzini & Tsamir (2001) and Piez & Voxman (1997) students need to be introduced to a variety of representations of inequalities, for it other than a functions approach, sign chart methods was also introduced on the students. Sign chart methods need to be introduced through linking him with the function approach so that students do not only interpret the quadratic inequalities procedurally, which is in accordance with what is expressed by Tsamir & Bazinni (2004) that if students are just introduced to the procedures students will lose meaning on a variable and coefficient on a quadratic inequalities.</td></tr>
</table>

Figure 1. Hypothetical learning trajectories of Quadratic Inequality

To address these problems, there must be an alternative approach. Bazzini & Tsamir (2001) suggest a function approach for teaching quadratic inequality. Even Tsamir & Reshef

(2006), Tsamir & Almog (2001) say that students who use the function approach is more likely to give the right answer in solving quadratic inequality. The same thing is said by Heid and Usiskin (in Kieran 2004) and Kieran (2004) function is the core of algebra. Thus, in formulating hypothetical learning process (developmental progression of thinking and learning) and the learning activity (sequences of instructional tasks) the approach function is used. It is supported by an analysis of the history and principle of quadratic inequality. Function approach of inequality is closer to a concept conveyed by mathematician (Boero & Bazzini, 2004; Kieran, 2004). Based on the analysis above, the learning trajectories of didactical design of quadartic inequality are arranged (figure 1).

Implementation of Hypothetical Learning Trajectories

In metapedadidactical analysis stage, hypothetical learning trajectories which has been arranged then be implemented in the learning process. During the learning process, students are exposed to the didactic situation which is designed based on the learning trajectory. The students' response towards the learning is observed and the interview is conducted with the students.

On situation 1, students solve the problems by using trial and error. First, students arrange the geometric representation of the problem, then students try certain numbers which equal to 72 m^2 of the land size. After the students acquire land size i.e, 11 × 11 m^2, then the students immediately draw the conclusion that in order for a smaller sizes of building equal to 72 m^2, the smaller the size of the land must be equal to 11 m , as shown in the figure below:

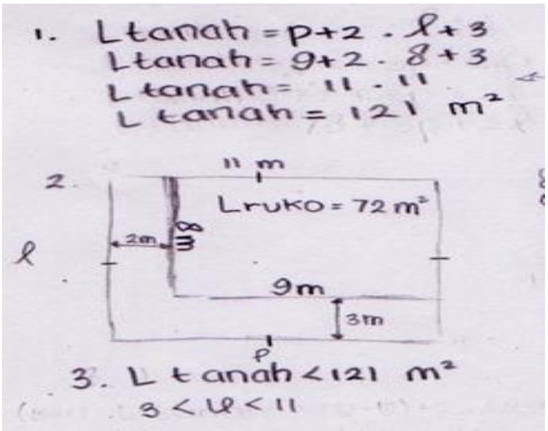


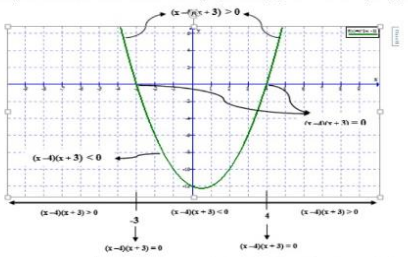
Figure 2. Student's response

This way of thinking is also used by students to solve the problems on situation 2. This indicates that most students do not immediately see the problems that are given as algebraic form, in this case quadratic inequality. Students first used arithmetic approach to solve the problems. This approach makes it difficult to tap into the flow path of learning through quadratic equations to quadratic function then to quadratic inequality. This is because the level of students' thinking is still in the process of arithmetic thinking. Therefore, it will be difficult to enter a learning trajectory made which learns the algebra directly (quadratic equation). Students are able to follow the path of learning based on the scaffolding given by the teacher. On situation 3 to situation 5, students are able to form a symbolic representation of the algebra problems. Students' ability is influenced by the scaffolding provided by the teacher on the situation 1 and situation 2.

In addition, the students also have difficulty in changing the quadratic equation to a quadratic function. This difficulty can be seen from the students' responses who do not understand which the function that will be drawn in graph. The first student wrote $(x - 2)(x - 3) = 72$ then change it into $x^2 - 5x - 66 = 0$, but the student was unable to see that the form was the same as $f(x) = x^2 - 5x - 66$. It is easier for the student to follow the path through a quadratic function first and then to a quadratic equation. This can be seen when the student was asked to define the broad functions of the shop beforehand, the student found it easier to determine quadratic equations and inequality as a representation of a given situation. On situation 3 to situation 5, students no longer perform trial and error; the students are able to compose an algebraic representation of quadratic inequality of the given problem. Students also can see the functions, drawing graphs and determine the set of solution from graph. At situation-5 students are expected to find a sign-chart method to solve quadratic inequality and find some help because the learning trajectory that passed before was inequality quadratic using function approach. The Students' difficulties just lie on the situations which are arranged a bit confusing so teacher must provide scaffolding. Even so, for the learning path students are able to follow it.

Learning Trajectory

Students' response in the implementation of hypothetical learning trajectories will be analyzed in restropective analysis stage. Based on the analysis of the students' responses, it shows that the path traversed by the students in learning quadratic inequality not always follow hypothetical learning trajectory. Therefore, hypothetical learning trajectory will be revised according to the students' responses. The revised section is the students' level of thinking that is still in arithmetic thinking leads to the bridging from arithmetic to algebra. Similarly, the trajectory passes from the quadratic equation to quadratic functions makes it difficult for students to determine which function is requested, which connects to the graphical representation. Therefore, the results obtained from the hypothetical learning trajectory revision shown as follows (revised part will be added with "**Revision**")

<table>
<tr><th>Developmental progression</th><th>Sequences of instructional tasks</th></tr>
<tr><td>Quadratic Equation</td><td>Situation-1
A shareholder wants to build a shop on a square-shaped land. That share holder sets aside 2 and 3 metres long land located beside and in front of the shop's yard.
1. What is the area of the land needed by the share holder if he/she plans to have a building area of 72 m^2?
2. That shareholder changes his/her mind because he/she limits the area for not exceeding 72 m^2. What is the area that he/she must have?</td></tr>
<tr><td>Quadratic function and quadratic function graphic</td><td>3. Determine the function which shows the relation between land area and shop area. (Revision)
4. Draw a graphic from that function. (Revision)
5. In number 4, you have determined how to determine land size (value x) from quadratic function graphic for shop area (72 cm^2). Now, show through the functional graphic that the land size (value x) generates an area of less than 72 cm^2 as the shareholder wants. (Revision)
6. Based on number 1-5, arrange how to determine the solution of quadratic inequality (land area function) from the graphic. (Revision)
7. According to number 6, determin the value x in the following equations.
a. $x^2 + 4x + 5 > 0$ (Revision)
b. $x^2 - 3x + 7 > 0$ (Revision)</td></tr>
<tr><td colspan="2">On Situation-1
From the observations found that student difficult to understand question number 2 (the question is given to encourage the occurrence of situations-formulations (construction of a quadratic inequality). The question "Describe and illustrate the graphical function that represents the area of the house to build?" confuses students because the word "describe and illustrate". Students do not understand the purpose and meaning of this word. This question was not to provide assistance so that students are able to formulate that the intersection graphs of functions with x-axis is land size (x value) for land area equal to 72 cm^2. For that question is replaced with guiding questions.</td></tr>
<tr><td>Quadratic inequality with real number solution x ($\forall x \in A, A \subset R$)</td><td>Situation-2
1. Determine the function which shows the relation between land area and shop area that you got from situation one.
2. Draw a graphic from that function. (Revision)
3. In situation one, you have showed the functional graphic of land size (value x) that generates a shop area of 72 cm^2 and for a shop area of less than 72 cm^2. Now, show a functional graphic that shows a shop area of more than 72 cm^2. (Revision)
4. According to number 1-5, determine the value x that fulfills following equations
a. $2x^2 + 5x + 2 > 0$ (Revision)
b. $-x^2 + 2x + 8 > 0$. (Revision)</td></tr>
<tr><td colspan="2">On Situation-2
Same as situation-1, in situation-2 students also difficulties to understand the meaning of question number 1. In learning with the situation-1 is indeed students no longer asks the purpose and meaning of the question "Describe and illustrate..." it is because teacher has explain it when learning situations-1. However, this question should be revised because of the difficulty students memaknainya if there are no explanations from the teacher.</td></tr>
<tr><td>Quadratic inequality with real number solution x ($\forall x \in R$)</td><td>Situation-3
Do you still remember the situation faced by the shareholder who wants to build a shop? He wanted to build a shop on a square-shaped land by setting aside his/her 2 and 3 metres long yard in the left and in front. However, that share holder changes his mind because he/she wants to buy land in the right side with the same size as the previous land and 70 m^2 land at the back. On the land which he/she buys, there will be a shop with the same area as the land.
What is the required size area of the earlier land in order to build the shop (the previous and the additional one) bigger than 72 m^2?
1. Determine the function which shows the relation between land area and shop area that you got from situation one. (Revision)
2. Draw a functional grapic that you found. (Revision)
3. Show according to the graphic of value x (land size) that suitable if the land area is bigger thatn 72 cm^2. (Revision)
4. Is it possible for a quadrtic inequalities that cannot be divided to have a set of solution? (Revision)
5. How to determine a set of solution of quadratic inequalities through its quadratic functional graphic?
6. Based on number 1-5, determine value x that fulfills following equations
a. $x^2 + 4x + 5 > 0$
b. $x^2 - 3x + 7 > 0$
c. $-x^2 + 2x - 6 < 0$ (Revision)</td></tr>
<tr><td colspan="2">On Situation-3
Students can understand this situation, because this situation is a continuation of the previous situation. The difficulty students is when determining the value of x which satisfy the above graphics for fuse-x (y > 0). The questions provided are not conceptually systematic, therefore the questions changed. So any questions number 4 and 5 as almost the same as the question of situation-2.</td></tr>
<tr><td>Quadratic inequality with no solution (solution x, $x \notin R$)</td><td>Situation-4
In situation one, a share holder wants to have a shop which is bigger than 72 m^2, and then he/she want to have a smaller area than that area.
Initially, the shareholder wants to build a shop on a square-shaped land. That share holder sets aside 2 and 3 metres long land located beside and in front of the shop's yard. However, he/she changes the plan beause he/she got additional fund. He/she wants to buy land in the right side with the same size as the previous land and 70 m^2 land at the back. On that land will be built a shop with the same area as the land.
What is the initial area that should be bought so that the whole area of the shop (initial and additional) is smaller than 72 m^2?
1. Determine the function which shows the relation between land area and shop area that you got from situation one. (Revision)
2. Draw a functional grapic that you found. (Revision)
3. Show according to the graphic of value x (land size) that suitable if the land area is less thatn 72 cm^2. (Revision)
4. Could it be possible for quadratic inequalities to have no solution? (Revision)
5. How to determine a set of solution of quadratic inequalities through its quadratic functional graphic?
6. Based on number 1-5, determine value x that fulfills following equations
a. $-x^2 + 3x - 7 > 0$
b. A squared-land has a with 5-x metres length and x metres wide. Determine all size of the land so that its area is bigger than 8 m^2.
c. $x^2 + 3x + 6 < 0$ (Revision)</td></tr>
<tr><td colspan="2">On Situation-4
Like situation-3, situation-4 didn't contains a quadratic inequalities problem that has no real numbers solution (the problem of the quadratic inequalities $ax^2 + bc + c < 0$ with $D < 0$ and $a > 0$). Based on this, the question "1. c. Specify the values of x which satisfy $x^2 + 3x + 6 < 0$" will be added in situation-4</td></tr>
<tr><td>Solving quadratic inequality by Sign Chart method</td><td>Situation-5
Pay attention to below functional graphic of $f(x) = x^2 - 5x - 6 = (x - 4)(x + 3)$:


1. From above illustration, what is the relation between the line number and the graphic? (Revision)
2. How if the graphic and y axis is omitted? Could we still be able to see which interval fulfills the proved equation? (Revision)
3. How to determine a set of solution by the line number without seeing the graphic? (Revision)
4. Arrange the steps to solve the quadratic inequation with the line number only. (Revision)</td></tr>
<tr><td colspan="2">On Situation-5
The students experience obstacles when dealing with this situation. Student confused the purpose of the picture. After the teacher provides scaffolding in the form of the question "what if the graph and axis-y is omitted? Whether we can still see the interval which satisfies the given inequality to on?" students can understand the situation. Based on this, the guiding question will be added in situation-5</td></tr>
</table>

Figure 3. Revision of Hypothetical Learning Trajectories of Quadratic Inequality

This revision will be the learning trajectory of quadratic inequality.

CONCLUSION

Based on the results of this study, it is gained an alternative learning trajectory, which is different from the learning trajectory that is used in the didactical design of the text book. Learning trajectory that is offered is learning quadratic inequality which start from the function approach. By starting the quadratic inequality through the function approach, it is expected that students understand quadratic inequality not only procedurally, such as the learning obstacles discovered by Tamba (2015). However, this study indicates that the analysis on the students level of thinking level and mathematical thinking tendency aren't analyzed further. Therefore, in designing the learning trajectories, the students' level of thinking and mathematical thinking must be a primary consideration because every student has different level of thinking and mathematical thinking orientation.

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