LEARNING TRAJECTORY OF QUADRATIC INEQUALITY

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ABSTRACT

A learning trajectory offers a description of key aspects in planning mathematics learning. It also helps teachers follow and interpret students’ mathematical thinking, so that learning can be developed in accordance with the characteristics of students, and even become a tool for teachers to develop curriculum. There are three main components of learning trajectory: learning goals, learning activities, and hypothetical learning processes. In this article, we constructed a learning trajectory of the quadratic inequality. This qualitative study used didactical design research with 105 grade 10 students as the participants. In the prospective analysis step, didactic design, learning obstacle, and quadratic inequality system were analyzed. Based on the results of this analysis, we constructed hypothetical learning trajectories in the form of didactical design. Then, hypothetical learning trajectories were implemented in the learning process. Student’s responses were analyzed qualitatively. Results of this analysis were used to revise the learning trajectory in order to obtain alternative trajectory learning outcomes of theoretical and empirical analysis. Finally, this article offers an alternative learning trajectory of quadratic inequalities that are different from the existing learning trajectories presented in the current textbook. The learning trajectory that is offered is the learning quadratic inequality which starts from the function approach.

Keywords: learning trajectory, quadratic inequality, didactical design research, learning obstacle

ABSTRAK

INTRODUCTION

Many researches have shown that Indonesian students’ mathematical thinking is low. A research (TIMSS) done by Mullis et al. (2016) confirmed the low mathematical thinking among Indonesian students. It showed that only 4% students who have good mathematical thinking, for instance in reasoning skill (high order thinking). The low mathematical thinking among Indonesian students is caused by the obstacle in learning specifically the quadratic inequality.

Inequality is one of the crucial topics in understanding various topics in mathematics such as algebra, trigonometry, and analytic geometry (Tsamir & Almog, 2001; Bazzini & Tsamir, 2001; Bicer, Capraro, & Capraro, 2014). Therefore, in designing the learning system which is proper to students' learning trajectories require an analysis of the developmental progression and student’s conceptual thinking on the quadratic inequality material. However, the existing didactical design in the classroom, tends not to perform the analysis on the thinking development and student’s conceptual on the material. It is shown that in all the didactical design used by teachers there is no difference in learning trajectory. While on Tamba’s research (2015) showed that students found some obstacles during the use of quadratic inequality through existing didactical design. Thus, it is important to do an empirical analysis to find out alternatives in the design of didactic learning trajectories quadratic inequality.

It is one of a Christian teacher’s responsibilities to thoroughly construct a learning design (didactic design) that is relevant to students’ uniqueness which is their learning trajectories. Van Brummelen (2009) argued that a teacher must arrange a learning design based on the continual analysis and reflection of their teaching experience.

LITERATURE REVIEW

In preparing the didactic design, teachers should consider about how students will go through the learning trajectory so the learning objectives can be achieved. Having these considerations, teacher is able to design the didactic situation in accordance with the students' learning trajectories. Simon (1995) used the hypothetical learning trajectories term first to show how teacher designs a learning.

Simon used the word "hypothetical" to indicate that part of the learning trajectories is flexible, where teacher can change the learning objectives and adapt the planning aspects based on the teacher’s perception on students understanding levels and teacher’s observation on performance of students while doing the tasks in the classroom. Therefore, the actual learning trajectories aren’t known before. According to Simon (1995), there are three main components of the learning trajectories: learning goals, learning activities and the hypothetical learning process. These are developed by Clements and Sarama (2004) into:
(1) learning goals; (2) developmental progression of thinking and learning, and (3) sequences of instructional tasks.

Constructing a didactical design in accordance with Hypothetical Learning Trajectories (HLT) is set by identifying the learning objectives for students—first component. Then, the didactical design is designed based on the learning objective. HLT design is based on the teacher’s knowledge of mathematics, teacher's knowledge of mathematics activities and representations, teacher’s hypothesis of student's knowledge, teacher's theories about mathematics learning and teaching, and teacher's knowledge of student’s learning of particular content.

To construct the learning trajectories, Simon and Tzur (2004) gave four principles that must be considered:

1. HLT is designed based on the students’ current mathematics knowledge.
2. HLT is a tool use for planning particular mathematical contents.
3. The exercises or the worksheets as tools to promote the learning on mathematics content are the key of the teaching process
4. Teachers must modify the HLT aspects if it’s not based on the students learning process continuously.

Therefore, to have learning trajectories that are match with students’ learning process, Hypothetical Learning Trajectories (HLT) must be implemented and revised according to students’s response in the classroom.

RESEARCH METHODOLOGY

This qualitative study used didactical design research. There are three steps in conducting this research (Suryadi, 2013) that are (1) situation didactical analysis before learning process in the form of a didactical design hypothesis (prospective analysis); (2) Metapedadidactical analysis, and (3) Retrospective analysis which relating the outcome of situation didactical analysis hypothesis and metapedadidactical analysis. In the prospective analysis step, current didactic design, learning obstacle, and quadratic inequality system were analyzed. This analysis was used to design hyphotetical learning trajectories. Then, in metapedidaktik stage the didactic design arranged from Hyphothetical Learning Trajectories was implemented in the learning process. Throughout this implementation process, all students’ responses to instructional taks were observed and interview about their difficulty. Finally, in prospective analysis stage, hyphotetical learning trajectories was modified based on the prior metapedadidaktik analysis. This study was conducted at ABC Senior High School, Bandung, and the participants of this research were 105 students of grade 10.

RESULT AND DISCUSSION

Hypothetical Learning Trajectories

The arrangement of Hypothetical Learning Trajectories is made in prosepective analysis stage through analyzing learning obstacle, current didactic design (textbook, lesson plan, curriculum), learning obstacle and quadratic inequality system (in mathematics’
context). According to this analysis result, hypothetical learning trajectories will be constructed. There are three components that must be considered in constructing a hypothetical learning trajectories: (1) learning goals; (2) developmental progression of thinking and learning; and (3) sequences of instructional tasks (the learning activity).

1. Learning Goals

Based on the mathematical knowledge of the students who have studied the quadratic equations, linear inequality and the learning objectives on the curriculum of senior high school (Kementrian Pendidikan dan Kebudayaan Republik Indonesia, 2013) then the purpose of the learning quadratic inequality are:

- Students are able to use the properties and rules about quadratic inequality
- Students are able to perform algebraic manipulations in the calculations associated with the quadratic inequality
- Students are able to design and to complete mathematical models of the problems related to quadratic inequality.

2. Developmental progression of thinking and learning and sequences of instructional tasks.

The arrangement of developmental progression of thinking and learning and sequences of instructional tasks is based on current didactic analysis (either used by teacher or research result and expert’s judgement), learning obstacle and mathematical knowledge about quadratic inequality.

From an analysis of textbook, lesson plan and students’ handout used by a teacher, several findings have been found as follow: (1) A teacher focuses solely on the number line approach and procedur of teaching quadratic inequality; (2) Quadratic inequality which is delivered by a teacher focuses only on manipulating algebra and the number line method; (3) The learning does not stress the difference between equation and inequality (no transition from equation and inequality). This approach causes an obstacle towards students’ learning.

Some obstacles faced by students caused by this approach are (1) Students face difficulty in dealing with quadratic inequality when the problem is presented in function or graphical representation; (2) a single approach (sign-chart method) that the students use cannot solve all quadratic inequality problems, and it only makes students undertand it procedurally (Tamba, 2015). The issue of learning obstacle is discussed in a research conducted by Tamba with the same students as a research subject in this study. Learning obstacles in this quadratic inequality system are (1) equation generalization towards inequality (students did not change the inequality symbol when multiplying quadratic inequality with negative number); (2) a generalization of two variables inequality to quadratic inequality (students wrongly understand the relation between graphical representation and quadratic inequality solution).
To address these problems, there must be an alternative approach. Bazzini & Tsamir (2001) suggest a function approach for teaching quadratic inequality. Even Tsamir & Reshef...
(2006), Tsamir & Almog (2001) say that students who use the function approach is more likely to give the right answer in solving quadratic inequality. The same thing is said by Heid and Usiskin (in Kieran 2004) and Kieran (2004) function is the core of algebra. Thus, in formulating hypothetical learning process (developmental progression of thinking and learning) and the learning activity (sequences of instructional tasks) the approach function is used. It is supported by an analysis of the history and principle of quadratic inequality. Function approach of inequality is closer to a concept conveyed by mathematician (Boero & Bazzini, 2004; Kieran, 2004). Based on the analysis above, the learning trajectories of didactical design of quadratic inequality are arranged (figure 1).

**Implementation of Hypothetical Learning Trajectories**

In metapedadidactical analysis stage, hypothetical learning trajectories which has been arranged then be implemented in the learning process. During the learning process, students are exposed to the didactic situation which is designed based on the learning trajectory. The students’ response towards the learning is observed and the interview is conducted with the students.

On situation 1, students solve the problems by using trial and error. First, students arrange the geometric representation of the problem, then students try certain numbers which equal to 72 m² of the land size. After the students acquire land size i.e, 11 × 11 m², then the students immediately draw the conclusion that in order for a smaller sizes of building equal to 72 m², the smaller the size of the land must be equal to 11 m, as shown in the figure below:

![Figure 2. Student’s response](image)

This way of thinking is also used by students to solve the problems on situation 2. This indicates that most students do not immediately see the problems that are given as algebraic form, in this case quadratic inequality. Students first used arithmetic approach to solve the problems. This approach makes it difficult to tap into the flow path of learning through quadratic equations to quadratic function then to quadratic inequality. This is because the level of students' thinking is still in the process of arithmetic thinking. Therefore, it will be difficult to enter a learning trajectory made which learns the algebra directly
(quadratic equation). Students are able to follow the path of learning based on the scaffolding given by the teacher. On situation 3 to situation 5, students are able to form a symbolic representation of the algebra problems. Students’ ability is influenced by the scaffolding provided by the teacher on the situation 1 and situation 2.

In addition, the students also have difficulty in changing the quadratic equation to a quadratic function. This difficulty can be seen from the students’ responses who do not understand which the function that will be drawn in graph. The first student wrote \((x − 2)(x − 3) = 72\) then change it into \(x^2 − 5x − 66 = 0\), but the student was unable to see that the form was the same as \(f(x) = x^2 − 5x − 66\). It is easier for the student to follow the path through a quadratic function first and then to a quadratic equation. This can be seen when the student was asked to define the broad functions of the shop beforehand, the student found it easier to determine quadratic equations and inequality as a representation of a given situation. On situation 3 to situation 5, students no longer perform trial and error; the students are able to compose an algebraic representation of quadratic inequality of the given problem. Students also can see the functions, drawing graphs and determine the set of solution from graph. At situation-5 students are expected to find a sign-chart method to solve quadratic inequality and find some help because the learning trajectory that passed before was inequality quadratic using function approach. The Students’ difficulties just lie on the situations which are arranged a bit confusing so teacher must provide scaffolding. Even so, for the learning path students are able to follow it.

**Learning Trajectory**

Students’ response in the implementation of hypothetical learning trajectories will be analyzed in restropective analysis stage. Based on the analysis of the students’ responses, it shows that the path traversed by the students in learning quadratic inequality not always follow hypothetical learning trajectory. Therefore, hypothetical learning trajectory will be revised according to the students’ responses. The revised section is the students’ level of thinking that is still in arithmetic thinking leads to the bridging from arithmetic to algebra. Similarly, the trajectory passes from the quadratic equation to quadratic functions makes it difficult for students to determine which function is requested, which connects to the graphical representation. Therefore, the results obtained from the hypothetical learning trajectory revision shown as follows (revised part will be added with “Revision”)
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Figure 3. Revision of Hypothetical Learning Trajectories of Quadratic Inequality

This revision will be the learning trajectory of quadratic inequality.
CONCLUSION

Based on the results of this study, it is gained an alternative learning trajectory, which is different from the learning trajectory that is used in the didactically designed text book. Learning trajectory that is offered is learning quadratic inequality which start from the function approach. By starting the quadratic inequality through the function approach, it is expected that students understand quadratic inequality not only procedurally, such as the learning obstacles discovered by Tamba (2015). However, this study indicates that the analysis on the students level of thinking and mathematical thinking tendency aren’t analyzed further. Therefore, in designing the learning trajectories, the students’ level of thinking and mathematical thinking must be a primary consideration because every student has different level of thinking and mathematical thinking orientation.

REFERENCES


