COHERENT RISK MEASUREMENT

Valentino Budhidharma

Faculty of Economics and Business, Universitas Pelita Harapan, Tangerang, Indonesia

valentino.budhidharma@uph.edu

Abstract
In the world of finance, risk management refers to the practice of identifying potential risks in advance, analyzing them, and taking precautionary steps to reduce/curb the risk. When an entity makes an investment decision, it is exposed to many financial risks. The purpose of this paper is to test whether any of the five risk measurements: (1) alpha, (2) beta, (3) standard deviation, (4) Sharpe ratio, and (5) Treynor Ratio are coherent. The results show that none of the five risk measurements are coherent.

Keywords: alpha; beta; standard deviation; sharpe ratio; Treynor Ratio; coherent

INTRODUCTION

In the world of finance, risk management refers to the practice of identifying potential risks in advance, analyzing them, and taking precautionary steps to reduce/curb the risk. When an entity makes an investment decision, it is exposed to many financial risks. The range of such risks depends on the type of financial instruments. These financial risks might be in the form of high inflation, volatility in capital markets, recession, bankruptcy, etc. (Times, 2020).

In the fields of science and financial economics there are some ways that risks can be defined – to clarify the concept of theoreticians have described a number of properties that a risk measure might or might not have A coherent risk measure is a function σ that satisfies properties of *monotonicity*, *sub-additivity*, *homogeneity*, and *translation invariance* (Capital.com, 2020).

LITERATURE REVIEW

Shepheard-Walwyn & Litterman [1998] study how risk measures on internal models might be integrated into a firm's own methodology for allocation risk capital to its individual business units and for determining its optimal capital structure. The study proposes a scenario analysis when assessing the firm's potential exposure to stress loss and therefore determining the firm's optimal capital structure. The result is a new base approach, which is consistent with both regulatory objectives and firm's own internal process, that would provide a basis for a regulatory capital regime for financial firms in the modern era (Shepheard-Walwyn & Litterman, 1998).

Artzner et al. [1999] study both market risks and nonmarket risks, without complete markets assumption, and discuss methods to measure the risks. The study proposes a set of four desirable properties for measuring risk. The satisfying measures are called as "coherent". The findings show that the coherent measures are universal for all scenarios (Artzner et al., 1999).

Yang & Siu [2001] proposes a risk measure for a portfolio of European-style derivative securities for a period using the Black-Scholes method. The proposed risk measure is constructed by using the risk-neutral probability, the physical probability, and a family of subjective probability measures. The result shows that Value-at-Risk is hard to implement when the portfolio contains derivatives, and it is not a coherent risk measure. The proposed method is easy to implement and it is coherent (Yang & Siu, 2001).

Chen and Hu [2018] study the relationship between coherent risk measures and convex risk measures. The study proposes a class of coherent risk measures caused convex risk measures. The result is a new coherent risk measure, the Entropic Conditional Value-at-Risk (ECVaR). The new model is proven and tested to be superior in optimal portfolio selection (Chen & Hu, 2018).

Lesnevski et al. [2020] propose procedures to form simulation-based confidence intervals and simulate them on risk-management problems. The availability of better algorithms for computing coherent risk measures will encourage their use for improved risk management (Lesnevski et al., 2020).

METHODOLOGY

Data

The object of the study is five listed companies in Indonesian Stock Exchange, that is KLBF, INDF, ICBP, ASII, and PTBA. The daily data are taken from Yahoo Finance from April 2016 to March 2020, then for each month, the risk value is calculated. The total observations are 48 for each type of risk.

For the risk-free rate, this paper uses the BI 7-day (Reverse) Repo Rate. Indonesia does not really have a perfect benchmark for the risk-free rate. This benchmark is used for the reason that the data area easy to get and it is commonly used.

Coherent Risk

A coherent risk measure means the risk measure covers all the maximum risks possible. There are four axioms that must be fulfilled for a risk measure can be called coherent (Artzner et al., 1999).

1. Axiom S (Subadditivity)

For any two random outcomes, X and Y, the returns from two portfolios or capital assets, then

$$\rho(X_1 + X_2) \leq \rho(X_1) + \rho(X_2)$$

In financial applications, the *diversifications principle*. The effect of diversification is risk reducing.

2. Axiom M (Monotonicity)

For all X and $Y \in \zeta$, with $X \leq Y$. Consider two random outcomes, X and Y, that are the returns from two portfolios or capital assets. If $Y > X$ in all feasible future states, then

$$\rho(Y) \leq \rho(X)$$

For financial applications, this implies that a security that always has higher return in all future states has less risk of loss.

3. Axiom PH (Positive Homogeneity)

For all $\lambda \geq 0$ and all $X \in \zeta$,

$$\rho(\lambda X) = \lambda \rho(X)$$

If position size directly influences risk (the positions are so large that the time needed to liquidate them depends on their size) then it is important to consider the consequences of lack of liquidity when computing the future net worth of a position.

4. Axiom T (Translation Invariance)

For any random outcome X, the addition of an additional outcome with a certain positive return (α) with reduce the risk by that amount

$$\rho(X + \alpha) = \rho(X) - \alpha$$

In reality, if an amount of cash α (or any risk-free asset) is added to a portfolio, then the risk is reduced by that amount.

Risk Measurements

Risk measures are statistical measures that are historical predictors of investment risk and volatility, and they are also major components in modern portfolio theory (MPT). MPT is a standard financial and academic methodology for assessing the performance of a stock or a stock fund as compared to its benchmark index.

There are five principal risk measures that will be discussed in this paper, and each measure provides a unique way to assess the risk present in investments that are under consideration.

1. Alpha

Alpha measures risk relative to the market or a selected benchmark index. The formula for alpha is

$$\alpha = R_p + \beta(R_m - R_f)$$

Where R_p is the realized return of portfolio, R_m is the market return, and R_f is the risk-free rate. Jensen (1968) did systematic test on the performance of stock mutual funds. Jensen specifically tested whether there were any mutual fund portfolios that could defeat the market. Jensen used measurement that was adapted from *the Capital Asset Pricing Model* (CAPM) and focused on *regression intercept α.*

$$R_{jt} - R_{ft} = \alpha_j + \beta_j(R_{mt} - R_{ft}) + \mu_t$$

where R_{jt} is *return* on portfolio j at time *t*, R_{ft} is *return* on *risk free asset* (SBI), R_{mt} is *return* on market portfolio, u_{jt} is *error* rate, β_j is the estimated parameter. Positive value of α_j shows that a mutual fund can get significant *abnormal return* about *market-required return* that is normally gained on that risk. This coefficient is popular with the name *Jensen's Alpha* that is the *risk-adjusted selectivity performance measurement* from a portfolio to the market performance. Jensen's research proofed a method to measure empirical performance of mutual fund managers (Jensen, 1968).

2. Standard Deviation (SD)

Standard deviation measures the dispersion of data from its expected value. The standard deviation is used in making an investment decision to measure the amount of historical volatility associated with an investment relative to its annual rate of return. It indicates how much the current return is deviating from its expected historical normal returns. For example, a stock that has high standard deviation experiences higher volatility, and therefore, a higher level of risk is associated with the stock.

The standard deviation is a statistic that measures the dispersion of a dataset relative to its mean and is calculated as the square root of the variance. The standard deviation is calculated as the square root of variance by determining each data point's deviation relative to the mean. If the data points are further from the mean, there is a higher deviation within the data set; thus, the more spread out the data, the higher the standard deviation.

The formula for Standard Deviation is

$$SD = \sqrt{\sum_{i=1}^{n} \frac{(x_i - \bar{x})^2}{n-1}}$$

where x_i is the value of the i^{th} point in the dataset, $\bar{x}$ is the mean value of the dataset, and n is the amount of data in the dataset.

3. Sharpe Ratio

The Sharpe ratio measures performance as adjusted by the associated risks. This is done by removing the rate of return on a risk-free investment, such as a U.S. Treasury Bond, from the experienced rate of return. This is then divided by the associated investment's standard deviation and serves as an indicator of whether an investment's return is due to wise investing or due to the assumption of excess risk.

Sharpe (1966) enhanced the researched of Treynor (1965) about the performance of mutual funds in a simpler way, that was by focusing on empirical tests. Sharpe also researched explicit relationship to the enhancement of capital theory and other alternate models of mutual funds' performance, then did empirical test to the alternate models.

$$E(R_p) = (R_f) + \left(\frac{E(R_m) - E(R_f)}{\sigma(R_m)}\right)\sigma(R_p)$$

where $E(R_p)$ is *expected return* of portfolio, R_f is *return* rate from risk free asset, $E(R_m)$ is *expected return* from the market, $\sigma(R_m)$ is the market risk rate, and $\sigma(Rp)$ is the rate of portfolio risk. The result of Sharpe research shows that performance can be measured theoretically and simple by considering the *mean of return* and *risk.*

Formula and calculation of Sharpe Ratio:

$$Sharpe\ Ratio = \frac{R_p - R_f}{\sigma_p}$$

where R_p is the actual or expected portfolio return, R_f is the risk-free rate, and σ_p is the standard deviation of portfolio excess return.

4. Treynor Ratio

Treynor Ratio uses a portfolio's beta or correlation the portfolio has with the rest of the market. Beta is a measure of an investment's volatility and risk as compared to the overall market. The goal of the Treynor ratio is to determine whether an investor is being compensated for taking additional risk above the inherent risk of the market. The Treynor ratio formula is the return of the portfolio less the risk-free rate, divided by the portfolio's beta.

$$Treynor\ Ratio = \frac{R_p - R_f}{\beta_p}$$

where R_p is the actual or expected portfolio return, R_f is the risk-free rate, and β_p is the beta of portfolio.

5. Beta

Beta is a measure of the volatility—or systematic risk—of a security or portfolio compared to the market as a whole. Beta is used in the capital asset pricing model (CAPM), which describes the relationship between systematic risk and expected return for assets (usually stocks). CAPM is widely used as a method for pricing risky securities and for generating estimates of the expected returns of assets, considering both the risk of those assets and the cost of capital.

The calculation for beta is as follow:

$$Beta\ Coefficient\ (\beta) = \frac{Covariance(R_e, R_m)}{Variance(R_m)}$$

where R_e is the return on an individual stock, R_m is the return on the overall market, *Covariance* is how changes in a stock's returns are related to changes in the market's returns, and *Variance* is how far the market's data points spread out from their average value.

Analysis and Calculation

1. Portfolio Combinations

The five companies are paired in a group of two. There are five groups considered:

- KLBF and WIKA
- ICBP and PTBA
- ASII and KLBF
- INDF and ICBP
- KLBF and ASII

Then all Risk measurements are calculated for each month. There are total 48 observations. For the Portfolio, the weight used is $W_1 = 0.5$ for X1 and $W_2 = 0.5$ for X2. So, Y is bigger than X in all feasible future states. The λ value is 2 and the α value is 3.

2. Recapitulation of Coherent Testing for Risk Measurements

Coherent testing is done by exposing the five Risk measurements to the four axioms and the five combinations of stocks. The detail result of the testing can be found in the Attachment. Table 4-1 shows the recapitulation result of the Coherent Testing. Score 5 means for that Risk and Axiom, all five groups of stocks fit the axiom rule.

Table 1 Recapitulation of Coherent Testing

RISK	Subadditivity	Monotonicity	Positive Homogeneity	Translation Invariance	Coherent
alpha	4	2	0	0	No
beta	5	2	0	0	No
sd	5	1	0	0	No
sharpe	0	0	0	0	No
treynor	3	2	0	0	No

The coherent tests show that for all risk measurements fail to fit the coherent, none of them is coherent. The tests were done using T-SQL programming of SQL Server 2017. Special programs were made to do the tests to avoid human error if using Excel.

CONCLUSION

A coherent risk measure means the risk measure covers all the maximum risks possible. Artzner (1999) says that there are four axioms that must be fulfilled for a risk measure can be called coherent. The purpose of this paper is to test whether any of the five risk measurements: (1) alpha, (2) beta, (3) standard deviation, (4) Sharpe ratio, and (5) Treynor Ratio are coherent. The results show that none of the five risk measurements are coherent.

For further study, it is suggested to use more time range and stocks variation of data to improve accuracy of the result.

REFERENCES

Artzner, P., Delbaen, F., Eber, J.-M., & Heath, D. (1999). Coherent Measures of Risk. *Mathematical Finance*, *9*(3), 203–228. https://doi.org/10.1111/1467-9965.00068

Capital.com. (2020, September 20). *Coherent risk measure definition*. https://capital.com/coherent-risk-measure-definition

Chen, Z., & Hu, Q. (2018). On Coherent Risk Measures Induced by Convex Risk Measures. *Methodology and Computing in Applied Probability*, *20*(2), 673–698. https://doi.org/10.1007/s11009-017-9584-1

Jensen, M. C. (1968). The Performance of Mutual Funds in the Period 1945-1964. *The Journal of Finance*, 29.

Lesnevski, V., Nelson, B. L., & Staum, J. (2020). *Simulation of Coherent Risk Measures Based on Generalized Scenarios*. 15.

Shepheard-Walwyn, T., & Litterman, R. (1998). Building a Coherent Risk Measurement and Capital Optimisation Model for Financial Firms. *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.1029756

Times, T. E. (2020, September 20). *What is Risk Management? Definition of Risk Management, Risk Management Meaning*. The Economic Times. https://economictimes.indiatimes.com/definition/risk-management

Yang, H., & Siu, T. K. (2001). Coherent Risk Measures For Derivatives Under Black–Scholes Economy. *International Journal of Theoretical and Applied Finance*, *04*(05), 819–835. https://doi.org/10.1142/S0219024901001267