

STUDENTS' WAYS OF UNDERSTANDING AND THINKING BASED ON HAREL'S THEORY IN SOLVING SET PROBLEMS

Christina Monika Samosir¹, Tatang Herman^{2*}
^{1,2}Universitas Pendidikan Indonesia, Bandung, JAWA BARAT

Correspondence Email: tatangherman@upi.edu

ABSTRACT

Ways of thinking (WoT) and Ways of Understanding (WoU) are important in learning mathematics. This study aims to analyze the characteristics of students' ways of thinking (WoT) and ways of understanding (WoU) when solving set problems. The participants in this study were 27 students in the seventh grade at a private secondary school in Bandung. Four students were selected as research subjects. The research method used is qualitative research. The instruments in this study were the researchers and the set material problem-solving test. The data collection technique in this study was carried out through a test, interviews, and documentation. The research results showed that the majority of students who were correct in their way of thinking were also correct in their way of understanding. And the majority of students who have the incorrect way of thinking also have the incorrect way of understanding. The study's results also indicated that the majority of student errors stemmed from a lack of proper understanding of the question and the concept of set theory.

Keywords: Harel's theory, set problems, set theory, ways of thinking, ways of understanding

ABSTRAK

Cara berpikir dan cara memahami merupakan hal yang penting dalam belajar matematika. Penelitian ini bertujuan untuk menganalisis karakteristik cara berpikir dan cara memahami siswa dalam menyelesaikan masalah himpunan. Partisipan pada penelitian ini adalah siswa kelas VII di salah satu SMP Swasta di Bandung berjumlah 27 partisipan. Empat siswa dipilih sebagai subjek penelitian. Metode penelitian yang digunakan adalah penelitian kualitatif. Instrumen dalam penelitian ini adalah peneliti sendiri dan tes pemecahan masalah materi himpunan. Teknik pengumpulan data dalam penelitian ini dilakukan dengan memberikan tes, wawancara dan dokumentasi. Hasil penelitian menunjukkan bahwa mayoritas siswa yang memiliki cara berpikir yang benar juga memiliki cara pemahaman yang benar. Dan mayoritas siswa yang memiliki cara berpikir yang salah juga memiliki cara pemahaman yang salah. Hasil penelitian juga menunjukkan bahwa sebagian besar kesalahan siswa disebabkan karena siswa kurang memahami soal dan konsep materi himpunan dengan baik.

Kata Kunci: teori harel, permasalahan himpunan, teori himpunan, cara berpikir, cara memahami

INTRODUCTION

Mathematics is a science that emphasizes the formation of thinking skills (Samo et al., 2017). Thinking is a process that involves mental activity because there is a problem that is being thought about and finding a solution (Nggaba, 2020). The results can be in the form of ideas, thoughts, or decisions. Representing, defining, interpreting, calculating, concluding, guessing, proving, symbolizing, composing, changing, applying, generalizing, modeling, predicting, connecting, formulating, searching, classifying, anticipating, and problem solving are some examples of mental activity (Harel, 2013). The purpose of thinking is to find solutions or answers to problems and make the right decisions (Hester, 2021). When students form ways of thinking, it allows students to form ways of understanding to solve mathematical problems (Jayanti et al., 2018). According to Harel, learning mathematics in a classroom setting is meant to foster this way of thinking and understanding (Harel, 2013).

Students' thinking processes differ because they are influenced by different levels of students' mathematical abilities, the schemas that are formed, and different understandings of a concept (Setyaningsih et al., 2018). To form a good way of thinking and understanding for students, a teacher must have competence in managing the teaching and learning process in both the material and in increasing student activity and motivation in learning mathematics (Watson & Harel, 2013). When students acquire a proficient way of thinking, it enables them to acquire the capacity to understand concepts during the cognitive process of resolving mathematical problems. Understanding stands as a crucial foundation for pondering the correlation between each element in scenarios involving a dynamic mentality. This understanding represents a mathematical process that should be understood by every student.

One of the mathematical topics that students often find difficult to solve is set theory. (Aulia et al., 2022; Ratnasari et al., 2019; Sundari, R., Andhany, E., & Dur, 2019; Walingkas & Sulangi, 2022). "The topic of sets has a fairly high level of difficulty, especially in the case of word problems, leading many students to experience difficulties in solving set-related word problems."

This research is useful to see the relationship between the way of thinking and the way of understanding. This research was also conducted to see students' thinking processes in working on problems on set topic. By knowing the students' thinking processes, the teacher knows the weaknesses of students in working on set problems. Thus, the teacher can design appropriate learning by considering this.

Based on the background above, the researcher is motivated to analyze the characteristics of the Ways of Thinking and also Ways of Understanding students in solving mathematical problems, especially set material based on Harel's Theory. The limitation of this research is that this research is not to look at ways of thinking and understanding in understanding a concept about set material. However, this research specifically only looks at ways of thinking and understanding in solving a problem about the set topic.

LITERATURE REVIEW

Harel's Theory

In the context of mathematics education, "ways of thinking" and "ways of understanding" refer to two crucial aspects that influence how students process, comprehend, and solve mathematical problems. "Ways of Thinking" refers to the strategies used by students to solve mathematical problems. Students can have various ways of thinking when faced with a problem. "Ways of Understanding" refer to the level of comprehension that students have about mathematical concepts. Students can have a deep and profound understanding of a concept or only a superficial understanding. Harel defines mathematics as a combination of ways of thinking and also ways of understanding, as follows. "Mathematics is a combination of two sets: The first set is a collection or structure consisting of particular axioms, definitions, theorems, proofs, problems, and solutions. This subset consists of all the institutionalized ways of understanding mathematics throughout history. The next set consists of all the ways of thinking that are characteristics of the mental acts whose products made up the first set." (Harel, 2008b). Harel argues that ways of thinking are the characteristics of mental actions carried out by someone with ways of understanding which are the product.

Ways of thinking affect students' understanding of mathematical concepts and vice versa (Harel, 2008a). The first category, ways of understanding, consists of certain theorems, definitions, axioms, problems, proofs, and solutions. The next category, namely way of thinking, is a mental action that results in the form of the first category, namely way of understanding. Therefore the process of thinking and understanding is something that is interconnected.

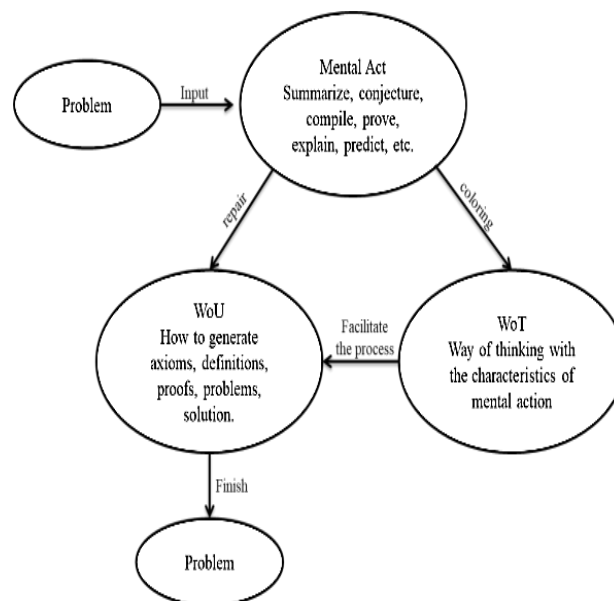


Figure 1. Patterns of Learning Mathematics

RESEARCH METHODOLOGY

This study is a qualitative triangulation with phenomenology. Phenomenology is a suitable methodology for gaining insight into the essence or structure of life experience (Walker, 2007). The research participants were 27 junior high school students in Bandung, seventh grade in the first semester, who received set material using the 2013 curriculum. Four students were chosen as research subjects for further interviews. The researchers selected them as research subjects because they encountered difficulties while solving set problems in the preliminary study conducted by the researchers. Additionally, these four subjects were chosen due to intriguing differences in their ways of thinking and understanding, which were deemed worthy of analysis. The main purpose of this study is to analyze the characteristics of students' ways of thinking (WoT) and ways of understanding (WoU) when solving a set problem. The qualitative research instrument is the researchers themselves. Furthermore, test questions are used to collect the required information. Data collected from non-tests in the form of preliminary studies, interviews and documentation used to describe the relationship between students' ways of thinking and students' ways of understanding and the factors that influence both.

The data collection technique in this study was carried out by providing a problem regarding sets, and students were asked to work independently to solve the problem. The form of the question is given in the form of an essay so that the researchers can examine the flow of students' thinking through the answers that have been given. The question is: "A sports group consists of 70 students. Thirty-five students like swimming, 30 students like basketball, and 14 students like both. Make a Venn diagram from this information and determine the number of students who dislike both." Before being given to students, the question was first validated by a mathematics education expert on content, construct, and face validity. After students were given question, the researchers chose four subjects who were able to represent and describe the answers of other students. The selected subject will be analyzed regarding their way of thinking and understanding through the results of tests and interview.

The researchers classified the 27 students' answers into each characteristic. The characteristics of the WoT and WoU in question are how students think and understand the problem provided, as shown in the table below.

Table 1. The characteristics of the ways of thinking and understanding

| Characteristics | | Way of Understanding | |
|-----------------|---------------|----------------------|---------------|
| Way of thinking | Category | Correct (C) | Incorrect (I) |
| | Correct (C) | CC | CI |
| | Incorrect (I) | IC | II |

Table 1 shows the four characteristics of ways of thinking and understanding, namely: correct way of thinking and understanding (CC), correct way of thinking with incorrect way

of understanding (CI), incorrect way of thinking with correct way of understanding (IC), and incorrect way of thinking and understanding (II).

Specific steps in data analysis were carried out before entering, while in, and after finishing in the field. In this study, the researchers conducted a preliminary study on the initial research, then tested written question related to way of thinking and understanding in set material problem. Then analyze the information and then interpret the data. Miles (Marufi et al., 2022) states that validity in qualitative research includes transferability, credibility, confirmation, and dependability. Validation serves the purpose of providing confidence to both researchers and readers that the research findings have a strong and reliable foundation.

DISCUSSION

Each characteristic of ways of thinking and understanding has specific indicators, as shown in the table below.

Table 2. Description of Each Characteristic

| No | Characteristics | Description |
|----|--|---|
| 1 | The correct way of thinking and understanding (CC) | Steps/thinking strategies using good, logical, and systematic algorithms in solving set problems with a good knowledge of sets. |
| 2 | The correct way of thinking with the incorrect way of understanding (CI) | Steps/thinking strategies that are used systematically and logically use correct operations but do not understand the concept of sets correctly. |
| 3 | The incorrect way of thinking with the correct way of understanding (IC) | The steps/thinking strategies used are wrong, illogical, and not thorough with the algorithm/calculation process, but they understand the concept of sets well. |
| 4 | The incorrect way of thinking and understanding (II) | The steps/thinking strategies used are illogical, wrong, wrong/not careful with the algorithm/calculation process, and they do not understand the concept of sets well. |

The percentage of students in each characteristic is shown in table 3 below.

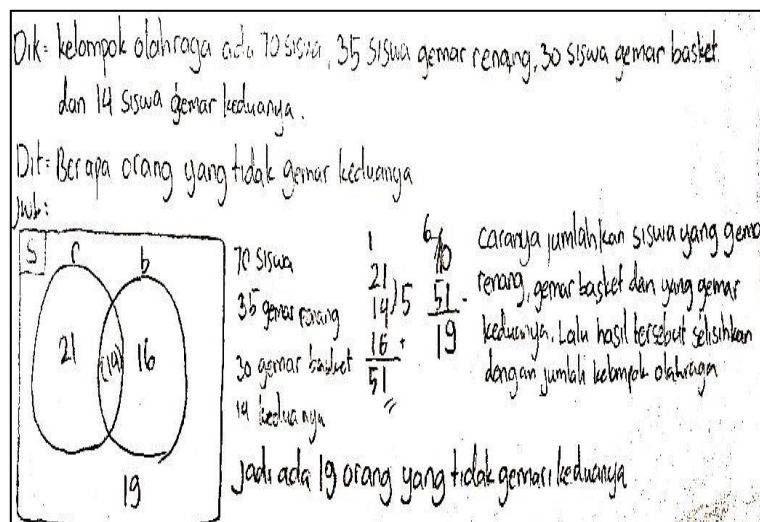
Table 3. The Percentage of Students in Each Characteristic

| No | Characteristic | Percentage |
|----|----------------|------------|
| 1 | CC | 37% |
| 2 | CI | 18.5% |
| 3 | IC | 14.8% |
| 4 | II | 29.6% |

From the findings, the number of students who belong to the characteristics of the correct way of thinking and understanding (CC) is 37%. The number of students who fall into the characteristics of the correct way of thinking with the incorrect way of understanding (CI) is 18.5%. The number of students who fall into the characteristics of the incorrect way of thinking with the correct way of understanding (IC) is 14.8%. The number of students who fall into the characteristics of the incorrect way of thinking and understanding (II) is 29.6%. To clarify how the WoT and WoU of each character are explained in more depth with the results of the following answers.

Student with the Correct Way of Thinking and Understanding (CC)

Students with this characteristic can think correctly, and students are also able to use the correct set concept in solving a given problem. No problems were found in the answers of students with this first characteristic. The following are the results of the student's solution with the correct WoT and correct WoU.



Translation:

It is known that there are 70 students in the sports group: 35 like swimming, 30 like basketball, and 14 like both.

Asked: How many people do not like both?

The trick is to add up the students who like swimming, basketball, and both. Then look for these results to differentiate them by the number of sports groups.

$$21 + 14 + 16 = 51$$

$$70 - 51 = 19$$

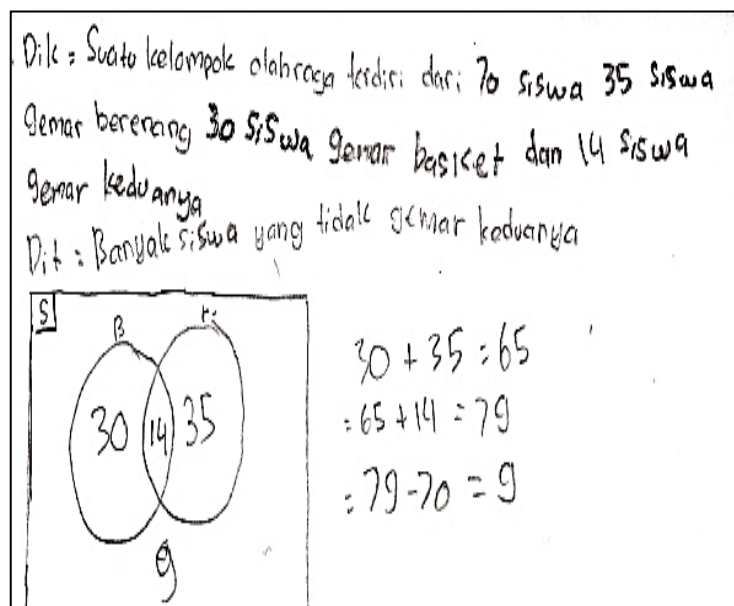
So there are 19 people who don't like either

Figure 2. The Student's Answer with CC Characteristic

Based on figure 2, the student already understood the intent of the question well, so the student can work on the question correctly, using good, systematic, and logical algorithms. In this case, the student wrote down the known elements and asked about the problem, then described the Venn diagram properly with a correct understanding of the set concept and solved the problem well. In answering this question, the student understood that there are as many people who "only like swimming" $35 - 14 = 21$ and who "only like basketball" are as many $30 - 14 = 16$ people. So the student can correctly find the number of people who dislike both. Thus this student is categorized as having the correct ways of thinking and understanding.

Student with the Correct Way of Thinking and Incorrect Way of Understanding (CI)

Students with this characteristic can think and complete operations correctly, but students have not been able to use the correct concept of sets. The following are the results of the answer of the student who has the correct WoT with the incorrect WoU.



Translation:

It is known that there are 70 students in the sports group: 35 like swimming, 30 like basketball, and 14 like both.

Asked: How many people do not like both?

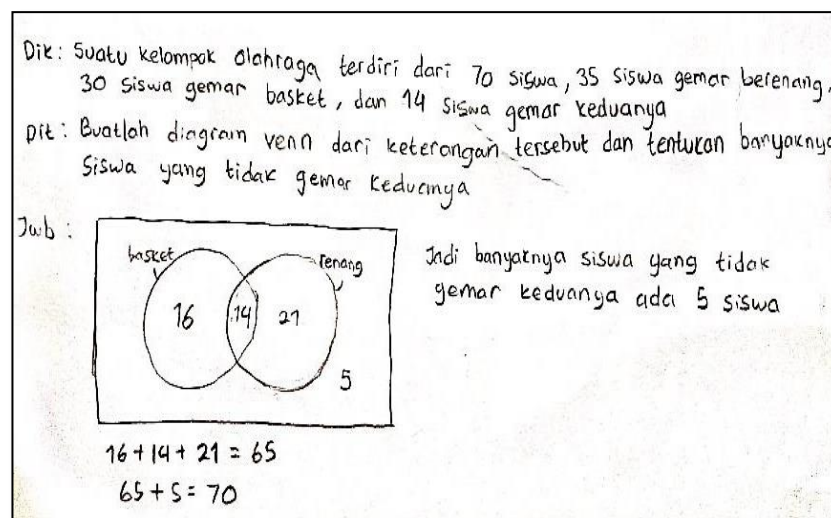
$$\begin{aligned}30 + 35 &= 65 \\65 + 14 &= 79 \\79 - 70 &= 9\end{aligned}$$

Figure 3. The Student's Answer with CI Characteristic

Based on Figure 3 and interview with the student, the student worked on question with good and correct calculations, but there are still misunderstandings about the concept of sets. In answering this question, the student did not understand that there were as many people who "only like swimming" $35 - 14 = 21$. The student, in this case, immediately wrote 35 on the Venn diagram for those who "only like swimming." Likewise, with those who "only like basketball," the student should have written $30 - 14 = 16$, but in this case, the student immediately wrote 30 on the Venn diagram for those who "only like basketball." It can be seen that the student did not have a good understanding of the problem given and concept of sets. Thus this student is categorized as having the correct ways of thinking and the incorrect ways of understanding.

Student with the Incorrect Way of Thinking and Correct Way of Understanding (IC)

Students with this characteristic have not been able to think and do calculations correctly, but students have been able to use the correct set concept in solving a given problem. The following are the result of the answer of the student who has the incorrect WoT with the correct WoU.



Translation:

It is known that there are 70 students in the sports group: 35 like swimming, 30 like basketball, and 14 like both.

Asked: How many people do not like both?

$$16 + 14 + 21 = 65$$

$$65 + 5 = 70$$

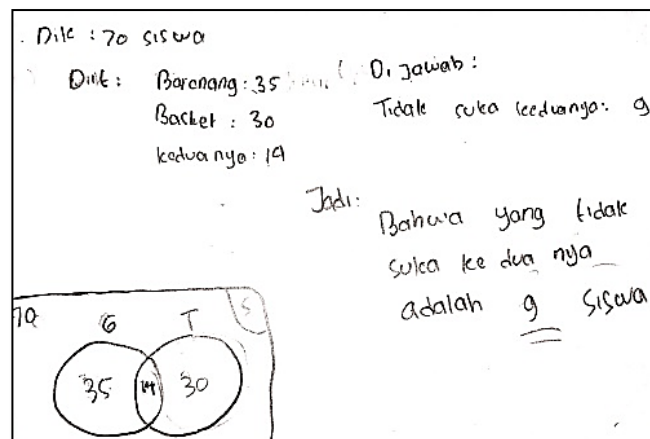
So there are 5 people who don't like either

Figure 4. The Student's Answer with IC Characteristic

Based on Figure 4, the student already understood the concept of set material, but the algorithm used was wrong, so the final decision was wrong. In answering this question, the student already understood that as many people "only like swimming" $35 - 14 = 21$. Students also understood that those who "only like basketball" are $30 - 14 = 16$ people. It can be seen that this student already had a good understanding of the concept of sets. However, in solving problems, this student still made mistakes in doing calculations. Students wrote $16 + 14 + 21 = 65$, even though the correct answer is 51. This is what caused the student to be wrong in the final decision. Thus this student is categorized as having the incorrect ways of thinking and the correct ways of understanding.

Student with the Incorrect Way of Thinking and Understanding (II).

Students with this characteristic have not been able to think correctly and have not been able to use the correct set concept in solving a given problem. The following are the results of a student's answers with the incorrect WoT and incorrect WoU.



Translation:
 70 students
 Swimming: 35
 Basketball: 30
 Both: 14
 So there are 9 people who don't like either

Figure 5. The Student's Answer with II Characteristic

Based on figure 5. The student could not complete logical steps, algorithms, and calculations and did not yet understand the concept of sets correctly. In solving this problem, the student did not present any calculation steps to get an answer. It can be seen that the student did not have a good understanding of the question and concept of sets. The student did not understand that as many people "only like swimming" $35 - 14 = 21$. The student, in this case, immediately wrote 35 on the Venn diagram for those who "only like

swimming." Likewise, with those who "only like basketball," students should have written $30 - 14 = 16$, but in this case, the student immediately wrote 30 on the Venn diagram for those who "only like basketball." Thus this student is categorized as having the incorrect ways of thinking and understanding.

Errors in answering set questions are generally due to students not understanding the concept and only reading without being interpreted. Errors also occur because students do not understand the problem and do not know how to draw a Venn diagram correctly. Students do not understand the concept of the intersection and union of two sets. The incorrect way of thinking and the incorrect way of understanding will have implications to the final result. The highest percentage is the correct way of thinking with the correct way of understanding (CC). Then followed by the characteristics of incorrect ways of thinking and incorrect ways of understanding (II). This shows that the majority of students who have the correct WoT also have the correct WoU. And the majority of students who have the incorrect WoT also have the incorrect WoU. This creates a two-way connection between way of thinking (WoT) and ways of understanding (WoU) in the principle of duality.

CONCLUSION

The researchers can conclude that the majority of students who have the correct way of thinking also have the correct way of understanding. And the majority of students who have the incorrect way of thinking also have the incorrect way of understanding. This creates a two-way connection between way of thinking (WoT) and ways of understanding (WoU) in the principle of duality in solving problems. The results of the study also showed that the majority of student errors were due to students not understanding the question and the concept of the set material properly.

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